

Isospin violation in threshold πN scattering

T. E. O. Ericson

Physics Department, Cern, CH-1211 Geneva 23, Switzerland

We discuss the electromagnetic corrections to the πN scattering lengths generated by minimal e. m. coupling from a knowledge of the low energy expansion of the πN elastic scattering amplitude as well as from the nucleon and Δ pole terms, all taken for purely strong interactions. We assume the heavy baryon limit; the e. m. and axial form factors are assumed to have the empirical values as well as the mass differences and the pion mass, such that there is no free parameter. The different terms have a clear physical and intuitive origin. In particular, a large isoscalar term appears in the elastic charged-pion scattering lengths. We attempt a comparison the results from chiral effective field theory (EFT) and a physical interpretation of the empirical constants in that approach. The results are applied to the energy shift and width of the $\pi^- p$ atom.

1 Introduction

In the limit of purely strong interactions the πN scattering amplitudes at threshold are fundamental quantities which enter into the discussion of various problems. They provide, for example, a basic test of the Tomozawa-Weinberg chiral relation for the isoscalar and isovector scattering lengths in the limit $m_\pi = 0$

$$a^- = \omega / (8\pi F_\pi^2) \simeq 0.089 m_\pi^{-1}; \quad a^+ = 0, \quad (1)$$

where $F_\pi = 93 \text{ MeV}$ is the pion decay constant[1, 2] and $\omega = m_\pi$ at threshold. The empirical isovector scattering length is the main ingredient and uncertainty in the forward dispersion relation, by which the πNN coupling constant 14.11 ± 0.05 is determined[3] etc.. The major precision source for these quantities are the remarkable measurements of the 1s level shifts in pionic hydrogen and deuterium as well as the corresponding widths. In the case of pionic hydrogen, present experiments have, or will shortly achieve, a precision of 0.2% for the shift and 1% for the width [4, 5] and these quantities convert in principle to similar precision for the scattering amplitudes.

There are 2 major ways by which one can approach the problem of determining the π^-p scattering length from pionic hydrogen data. The first is chiral effective field theory (EFT) [6, 7, 8]. Here one starts from an effective Lagrangian in the chiral limit and makes first a systematic expansion in orders $\mathcal{O}(p^n)$ of momentum corrections. The e. m. contribution to order α and the strong symmetry breaking are then expanded as small additional corrections. This approach introduces empirical constants to summarize short range contributions and these must be determined from experiments.

The other major approach, which I will follow below, is complementary and less ambitious [9, 10]. It starts from the empirical low energy expansion of the strong interaction amplitude in terms of energy and momentum. It is rather well explored including its principal dynamic features, although the experimental value of the scattering length needs further tuning. This system has well defined e. m. and axial form factors of the pion and nucleon (Δ) which are experimentally known as are the physical masses. Starting from this knowledge, we determine the e. m. corrections. The main guiding tool is minimal e. m. coupling or, in other words, current conservation. In addition, we use the same minimal coupling principle to determine the dispersive contributions from the radiative capture processes dominated by the nucleon and Δ isobar intermediate states. For this last problem, it is a considerable simplification to work in the heavy baryon limit and we will do so below. This approximation is also used in the EFT approach.

The objective of this second approach is to obtain an intuitive picture of the corrections and the physical mechanisms of isospin breaking. It will become clear that kinematic considerations in the wide sense is the key to several of the effects.

The plan is the following. I first discuss the nature and physics of the corrections to the leading order Deser-Trueman formula [11, 12], which relates the energy shift to the scattering length. I then discuss 'inner' e. m. corrections to the s wave scattering of charged pions from the nucleon at threshold and show that the dominant term is intimately connected to the well established p-wave πN scattering physics and the Δ isobar. This contribution can be summarized by the EFT chiral parameter f_1 [6, 7, 13]. Finally, I make a tentative comparison with the results of the EFT approach.

1.1 Step 1: How to get scattering lengths from atomic energy shifts: removal of the external Coulomb field

The s-wave threshold amplitude for the strong interaction has in the case of a single channel the low energy expansion

$$tg\delta = a_h + b_h q^2 + \mathcal{O}(q^2) \quad (2)$$

where q is the momentum and a_h, b_h are the scattering length and the range term respectively. Such an expansion does not by itself assume isospin invariance, but it means implicitly a short ranged interaction.

The atom is highly non-relativistic. To leading order the strong interaction shift is obtained using the $1s$ wave function for the point Coulomb field

$$\phi_{Bohr}(r) = \phi_{Bohr}(0) \exp(-\alpha mr) \simeq \phi_{Bohr}(0)(1 - \alpha mr + \dots), \quad (3)$$

where m is the reduced mass and $\alpha m r \ll 1$ over the strong interaction region. If we assume that the amplitude of Eq. (2) results from a short ranged pseudo-potential ('effective interaction Lagrangian'), the leading order $1s$ strong interaction energy shift is simply the Born approximation term:

$$\epsilon_{1s} = -\frac{4\pi}{2m}\phi(0)^2 a_h. \quad (4)$$

This is the time-honored standard reference shift referred to as the Deser-Trueman formula [11, 12] and there are of course corresponding expressions for any ns state. We now generalize this expression.

-Assume that the Coulomb field without strong interactions to be generated by the extended charge distributions of the pion and the proton. The corresponding e. m. form factor $F_{em}(q^2) = F_\pi(q^2)F_p(q^2)$ is experimentally known. Its origin is in principle irrelevant, although the issue of a Dirac versus a Pauli form factor of the proton must be faced later, when we compare to results of EFT in the heavy baryon limit. At this stage the Pauli form factor is the relevant one.

-This problem is highly non-relativistic and well described using wave functions. Outside the extended charge, the exact solution is the Whittaker function.

-In the limit that the strong interaction (2) has a negligible range compared to that of the charge distribution, the corresponding strong interaction energy shift has an exact solution [9].

-For the energy dependence of the scattering length we use the 'minimal e. m. coupling principle' or gauge invariance and replace the energy of a charged particle $\omega \rightarrow \omega - eV_C(r)$. Another way of saying the same thing is that one should locally at the interaction point use the correct interaction energy in the Coulomb potential.

There are now 3 separate physical effects [9]. First, the starting wave function at the origin is improved using the Coulomb potential corresponding to the joint charged distribution of the pion and proton, which is simpler to handle than the singular point Coulomb interaction. Second, the interaction does not correspond to the free scattering at threshold of Eq.(2), but corresponds to the energy shifted to that of the Coulomb potential at the interaction. Third, to second order in the scattering length, the binding gives characteristic cusp term which is nearly independent of assumptions. Finally, we always include tacitly a small, model independent vacuum polarization correction, which is conceptually irrelevant in the present context.

The extended charge changes the Coulomb wave function, chopping off the singular linear behavior of the wave function near the origin, which comes from the $1/r$ behavior of the Coulomb interaction. The wave function then varies normally with r^2 at the origin. The wave function at the origin is then

$$\phi_{Bohr}(0) \rightarrow \phi_{Bohr}(0)[1 - \alpha m \langle r \rangle_{em} + \mathcal{O}(\alpha^2)] \quad (5)$$

Here $\langle r \rangle_{em}$ is the expectation value over the extended charge density. The corresponding change in the energy shift is

$$\delta\epsilon_{1s;wf} = -\frac{4\pi}{2m}\phi_{Bohr}(0)^2(-2\alpha m \langle r \rangle_{em}) a_h \simeq (-0.9\%) \epsilon_h \quad (6)$$

Table 1: Coulomb corrections in percent. The vacuum polarization contribution is included in the total correction. (From Ref.[9])

	Extended charge	Renormalization	Gauge term	Total
δ_{1s}	-0.853(8)	0.701(4)	-0.95(29)	-0.62(29)
δ_{Γ}	-0.427(4)	0.701(4)	0.50(23)	1.02(23)
$\delta_{\pi^+p \rightarrow \pi^+p}$	0.853(8)	0.72(5)	-1.71(29)	0.35(29)

Detailed numerical values are given in Table 1. The wave function modification is not specific to the atomic bound state problem. It has a near identical counterpart in the elastic π^+p scattering close to threshold, but the interaction is now repulsive. For a neutron target such Coulomb corrections are of course absent. we should change the sign of the correction term Eq. (5), but there is no change for a neutron target, of course.

The energy change in the scattering amplitude (2) depends only on minimal coupling, such that it happens for the scattering near threshold of a positive pion as well, but with opposite sign.

$$\omega \rightarrow \omega - e(1 + \tau_3)t_3V_C(r). \quad (7)$$

For a short ranged limit we must use the potential energy at the origin.

The range term b_h in Ref. (2) corresponds to the experimental isoscalar and isovector range terms $b^+ \pm b^-$ for $\pi^\mp p$ interactions with the corresponding π^-p energy shift:

$$\delta\epsilon_{1s,gauge} = -\frac{4\pi}{2m}\phi_{Bohr}(0)^2(-\alpha\langle\frac{1}{r}\rangle_{em})b_h^{\pi^-p} \simeq (-1.0\%) \epsilon_h. \quad (8)$$

We note first, that it does not matter, to leading order, whether this range term is derived in Eq. (2) from terms of the momentum q or from the energy ω : the result will be identical [14], for the interaction is nearly on the mass shell. Second, the range term b_h has its own physics and should not be taken to be proportional to the scattering length.

The third contribution is the cusp effect generated by rescattering to second order in the scattering length by the binding in the Coulomb potential. It depends only weakly on the form factors. Its physics is that the incident wave in the interaction is renormalized by the scattering (effective field effect) as is known in the present context since a long time. It is accurately

$$\delta\epsilon_{1s,renorm} = -8\pi\alpha a_h^2 [2 - \gamma + \log 2\alpha - \langle\log mr\rangle_{em}]\phi_{Bohr}(0)^2 \simeq (+0.7\%) \epsilon_h \quad (9)$$

These corrections are exact to order $\mathcal{O}(\alpha^2)$ in the limit of a short ranged strong interaction.

A crucial term is the second one corresponding to the energy shifted amplitude depending on the π^-p range parameter b_h . One might think that the main contribution would follow from the ω dependence of the dominant isovector Tomozawa-Weinberg scattering length (1), which indeed appears to generate corresponding terms to next-to-leading order in the EFT expansion [6, 8, 13]. This is not the case. This isovector range term is largely canceled by the nucleon pole term and the net effect is quite small. Instead the contribution is mainly generated by the isoscalar range term proportional to ω^2 . This term is proportional to ω .

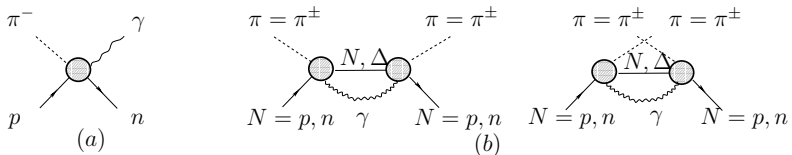


Figure 1: (a) the Kroll-Ruderman radiative capture process (b) diagrams of $\pi N \rightarrow X\gamma \rightarrow \pi N$ reactions generating isospin breaking in low-energy elastic πN scattering.

It is a fair question to ask how accurate the expression for this energy correction is in practice. This can be inferred in several ways. An easy and intuitive estimate is obtained modifying the level displacement expression of Eq. (4) taking the interaction density to have a form factor $F_{str}(r)$ instead of a point interaction. This means that the Coulomb interaction should be averaged over the interaction region folding in this additional form factor. The corresponding interaction shift becomes $\delta\epsilon_{1s,gauge} \propto \int F_{str}(q^2)F_\pi(q^2)G_p(q^2)/q^2 d^3q$. Using standard monopole and dipole form factors for the pion and proton with the scale of the ρ -meson and observing that the form factor of the Tomozawa-Weinberg term is also a monopole associated with the ρ -meson, indicates a typical small uncertainty from this source of about 0.15%, which is beyond present experimental uncertainties.

Up to this point we have not used the heavy baryon approximation. This means that for the proton charge form factor we should here use the Pauli form factor, which includes the $1/M_p$ terms from the magnetic charge density and not the Dirac form factor. More about that later.

All of these considerations are of course quite general and apply with small modifications for any hadronic atom. They are easily generalized to states ns of higher quantum number n as well as to the width. It is also easy to generalize the result to a repulsive Coulomb interaction such as π^+p , but the results will then concern the scattering length at threshold. Finally they can be generalized to the situation of two coupled channels [9]. The problem of extracting the hadronic scattering length from the atomic π^-p energy shift is therefore solved to a precision of about 0.1% on the level of the Coulombic interaction.

1.2 Step 2. Transverse photons; a dominant isospin breaking mechanism at threshold

The previous Coulombic contributions correspond to longitudinal photons. The question is then of the importance of transverse photons. In fact, these generate a large isoscalar correction, i. e., a term which is the same for all the elastic charged pion amplitudes as Dr. A. N. Ivanov and myself have recently shown [10].

A guide to the importance of such terms is the dominant contribution of the Kroll-Ruderman radiative capture process $\pi^-p \rightarrow \gamma n$ at threshold [15] (see Fig. 1a), which experimentally gives a $1s$ width of 8% of the strong interaction shift [16], a huge number. The term we consider is the corresponding dispersive shift (see Fig. 1b).

The matrix element for radiative capture can, for example, be derived from the Partially Conserved Axial Current (PCAC) relation using minimal e. m. coupling (or directly from

the nucleon pole term).

$$\partial_\mu A_\mu = -m_\pi^2 F_\pi^2 \phi_\pi(x); \quad \partial_\mu \rightarrow \partial_\mu \pm ie\mathcal{A}_\mu, \quad (10)$$

where \mathcal{A}_μ is the e. m. 4-vector potential. This statement corresponds to saying that we have electric dipole (E1) radiation due to the discontinuity in the current or, in other words, a kind of transition radiation. We calculate the dispersive contribution in the heavy baryon approximation. This is particularly convenient, since in this limit and at threshold the radiation comes only from the vertex itself in the Coulomb gauge and not from the pion and nucleon.

The characteristic features are:

- the transition is an axial one, such that its strength is well defined.
- it is natural to use the axial form factor $F_A(\vec{q}^2)$, which is empirically well approximated by a dipole shape (see, e. g., [17, 18])

$$F_A(\vec{q}^2) = (1 + \vec{q}^2/M_A^2)^{-2} \text{ with } M_A = (960 \pm 30) \text{ MeV} \quad (11)$$

- typical energy denominators $(p \pm m_\pi)^{-1}$ appear from intermediate states with the sign switching due to crossing.

-More precisely, the π^-p amplitude contribution gives a large nucleon isoscalar term of 3% in the limit $m_\pi = 0$. The contribution is for the π^-p case

$$\left(1 + \frac{m_\pi}{M_N}\right) \delta f_{1s}^{(n\gamma)} = \frac{3\alpha}{8\pi^2} \frac{g_A^2}{F_\pi^2} \mathcal{P} \int_0^\infty \frac{dp p F_A^2(p^2)}{p - m_\pi - i0}. \quad (12)$$

However, from the SU3 symmetry point of view, the Δ isobar and the nucleon are basically identical, but for the $N\Delta$ mass splitting and weight factors. It would be unnatural to include the nucleon only. In the limit of no $N\Delta$ mass splitting, the inclusion of Δ intermediate states gives a multiplicative factor 25/9 as compared to the nucleon term. This increases the previous 3% to 9%, an enormous correction!

Why is this so large? The reason is that the scale parameter is the axial mass M_A and not the pion one m_π which gives a factor 7 enhancement with respect to naïve expectations. When the $N\Delta$ mass splitting of $\approx 2m_\pi$ is brought in, it cuts the $\gamma\Delta$ term by 50% to a total isoscalar correction of about +6%. This is still very large, but we expect relativistic kinematic factors to cut it additionally to about 4.8%.

1.2.1 The case of $m_\pi \neq 0$

In this situation new characteristic terms appear of the type $cm_\pi \ln(m_\pi/M_A) + dm_\pi$, which are generated both from the nucleon and the Δ intermediate states. These terms depend only weakly on the exact value of M_A . In the particular case of the nucleon intermediate state, the term proportional to $m_\pi \ln m_\pi$ has the identical coefficient to the one found to third order in chiral EFT by Gasser et al. [8]. If we expand the amplitude $\delta f^{(n\gamma)}$ in Eq. (12) in terms of the small parameter $x = x_\pi = m_\pi/M_A$, we have in this case:

$$\left(1 + \frac{m_\pi}{M_N}\right) \delta f^{(n\gamma)} = \frac{3\alpha}{8\pi^2} \frac{g_A^2}{F_\pi^2} \left[\frac{5\pi}{32} M_A - m_\pi \left(\ln \frac{m_\pi}{M_A} + \frac{11}{12} + \mathcal{O}\left(\frac{m_\pi}{M_A}\right) \right) \right]. \quad (13)$$

Table 2: Contributions to the πN scattering lengths from dispersive radiative capture with nucleon and Δ intermediate states in units of $10^{-3} m_\pi^{-1}$ (from Ref. [10]).

$m_\pi = 0, \omega_\Delta = 0$	$(3.0(1)_{N\gamma} + 5.3(2)_{\Delta\gamma}) t_3^2$	$= 8.3(3) t_3^2$
$m_\pi = 0, \omega_\Delta \neq 0$	$(3.0(1)_{N\gamma} + 2.4(1)_{\Delta\gamma}) t_3^2$	$= 5.4(2) t_3^2$
$m_\pi \neq 0, \omega_\Delta = 0$	$(2.6(1)_{N\gamma} + 4.6(1)_{\Delta\gamma}) t_3^2 + (-0.8_{N\gamma} + 0.7_{\Delta\gamma}) t_3 \tau_3$	$= 7.2(2) t_3^2 - 0.1 t_3 \tau_3$
$m_\pi \neq 0, \omega_\Delta \neq 0$	$(2.6(1)_{N\gamma} + 2.5(1)_{\Delta\gamma}) t_3^2 + (-0.8_{N\gamma} + 0.3_{\Delta\gamma}) t_3 \tau_3$	$= 5.1(2) t_3^2 - 0.5 t_3 \tau_3$

When the Δ isobar is degenerate with the nucleon, this nucleon term strongly canceled by nearly a magnitude, such that the dependence on the pion mass becomes negligible. However, when the $N\Delta$ mass splitting is introduced in accordance with observation, the Δ contributions are quenched such that the contribution from the nucleon term is partially restored. A small term in the pion mass of about 50% of the value for the nucleon only survives. Numerical values for the different cases are given in Table 2.

1.3 Step 3. E. M. isospin violation and Chiral EFT

In the last few years a scheme has been developed to calculate strong and e. m. isospin violation to leading order for πN scattering using field theory methods based on an effective chiral Lagrangian [6, 7, 8, 13]. The amplitudes are calculated in a systematic expansion in powers of momenta (EFT). I will not enter into the details of this expansion, but only sketch a tentative comparison of some specific points. It is important to realize that certain of the predictions of such an effective field theory are specific and outside our present approach, while, on the contrary, our approach here generates terms of higher order in the EFT description than those presently considered. In the heavy baryon limit, the e. m. isospin breaking in the πN threshold amplitudes are related to the e. m. mass of the nucleon and the np e. m. mass difference in the EFT beyond the purely kinematic effects [6]. To next-to-leading order these relations can be expressed in terms of 3 constants $f_{1,2,3}$. In the case of $\pi^\pm p$ elastic scattering, which I chose for illustration, this gives the following relations.

$$M_n^{em} = -e^2 F_\pi^2 [f_1 + f_3] ; M_p^{em} = -e^2 F_\pi^2 [f_1 + f_2 + f_3] ; a_{\pi^\pm p}^{em} = -2\pi\alpha [f_1 \pm \frac{1}{4}f_2] \quad (14)$$

Following Ref. [6], I omit the term generated by the physical mass difference between the charged and neutral pion, which is of no concern in the present context.

It is of considerable interest to attempt to identify our results in the EFT expansion. Can we match our previous results to its explicit terms? A problem occurs in view of the basic difference with our approach. For example, in the discussion of the Coulombic terms, we start from the scattering amplitude and form factors as they would result in a description to all orders, but in the absence of e. m. interactions; in this sense we include physics not presently included in the EFT approach. We must therefore make sure we compare comparable quantities. First, we must use form factors in the heavy baryon limit. This means that we must use the Dirac form factor $F_p(q^2)$ and not the

Pauli form factor $G_p(q^2)$; the charge distribution generated by the magnetic moment of order M_N^{-1} should be omitted. We should also to this order omit the wave function correction $\alpha m \langle r \rangle_{em} a_{\pi-p}$ of Eq. (6), for the mass scale is set by the ρ -meson and it is of order $\alpha m/m_\rho a_h$ or formally of 4th order in the EFT expansion¹. The relevant term for the comparison is generated by Eq. (8). Here the dominant contribution comes from the isoscalar range term b^+ . It generates a contribution of order $m\alpha \langle r \rangle_{em} b^+$, which is of 4th order in the EFT expansion and outside the present EFT discussion; it will require an additional EFT constant. Similarly, the pole term in the isovector range term vanishes in the heavy baryon limit.

1.3.1 The EFT constant f_2 , the np mass difference and the Coulomb interaction

In the heavy nucleon limit, the constant f_2 in Eq. (14) describes the np e. m. mass splitting, which then results from the Coulomb self energy of the proton with the Dirac form factor: $(M_p - M_n)^{em} \propto f_2 \propto \frac{1}{2} \int d^3q [F_p(q^2)]^2/q^2$. The corresponding effect in the $\pi^c p$ scattering must be taken in the same limit with the Dirac factor. To leading order, the strong scattering amplitude is given by the Tomozawa-Weinberg term (1), which is linear in ω . The minimal coupling procedure generates an isospin-breaking contribution:

$$a_{\pi^\pm p}^{em} = \frac{1}{2}(1 + \tau_3)t_3^2 \frac{eV_C(0)}{(8\pi F_\pi^2)}, \quad (15)$$

where $eV_C(0) \propto \alpha \int F_\pi(q^2)F_p^2(q^2)/q^2 d^3q$. Compared to the proton e. m. self energy f_2 above, the similarity is striking and numerically the expressions are equal to about 15%. However, if the shape of the strong interaction is included with an interaction form factor, one obtains equality if

$$F_{str}(q^2)F_\pi(q^2)F_p(q^2) \simeq F_p^2(q^2) \quad (16)$$

or, since both the nucleon and pion form factors are governed by the ρ -meson mass

$$F_{str}(q^2) \simeq F_\pi(q^2) \simeq (1 + q^2/m_\rho^2)^{-1}.$$

We here obtain unexpectedly a quantitative result for the isovector interaction form factor. The Tomozawa-Weinberg term is frequently associated with a monopole interaction with a range given by the ρ meson mass $(1 + q^2/m_\rho^2)^{-1}$ [20], while both the nucleon and the pion have form factors closely approximated by a dipole, respectively a monopole, form factor with the ρ -meson mass.

This is suggestive, since it indicates that, indeed, the EFT f_2 coefficient comes from the Coulomb interaction. On the other hand, the EFT does not presently give the shape of the form factor. The last relation implies however that there is an intrinsic link between the proton, pion and the Tomozawa-Weinberg interaction form factors in EFT.

A proper account for the Coulomb interaction requires realistically that one goes beyond the present level of the EFT expansion and accounts for the Pauli form factor $G_{E,p}$ of the proton as well as for the terms generated by the isoscalar range term.

¹Since its coefficient is large, its order is unclear; its magnitude corresponds to 3rd order.

1.3.2 The EFT constant f_1 and the axial form factor

The dispersive contribution from intermediate $\gamma N(\Delta)$ states to the scattering length as given in Eq. (12) is isoscalar in the charged pion sector in the limit of a vanishing pion mass. This is exactly the symmetry property of the contribution in EFT by the next-to-leading order constant f_1 . This constant also appears as a part of the e. m. neutron mass M_n^{em} , but then it always comes in the combination $f_1 + f_3$ (see Eq.14)). These terms cannot be physically separated [6]. Here we have an interesting situation. The neutron e. m. mass is expected to be quite small compared to the np e. m. mass splitting of about 0.8 MeV and to be dominated by the magnetic self energy. It then vanishes in the heavy baryon limit with a likely uncertainty of $\pm(0.1 \sim 0.2) \text{ MeV}$ [21]. This corresponds to a value $F_\pi^2 |f_1| \simeq 1 \text{ MeV}$, while our value with the physical Δ isobar included gives $F_\pi^2 f_1 = -26(1) \text{ MeV}$, which is over a magnitude larger. Dimensional estimates inside of EFT give intermediate estimates $F_\pi^2 |f_1| = 6 \text{ MeV}$ and 12 MeV [6, 8]. There appears therefore numerically to be little relation of this parameter with the neutron e. m. mass, which suggests a massive cancellation between the EFT constants f_1 and f_3 . Such a cancellation occurs explicitly in a model evaluation inside a heavy quark model [22], which has certain similarities with our description. The connection of the constant f_1 to the nucleon e. m. mass is therefore tenuous and of little practical importance.

2 Conclusion

Isospin violation in πN elastic scattering has been shown here to have well determined contributions originating in the Coulomb field of the extended charge with little model dependence. These corrections are general and involve terms beyond present EFT approaches. To leading order in the strong interactions corresponding terms have counterparts in EFT in the heavy baryon approximation. In addition, terms with $\gamma N(\Delta)$ intermediate states give rise to important isoscalar isospin breaking terms which can be looked at as model descriptions of the unknown EFT constant f_1 . For a non-vanishing pion mass, the same mechanism generates small, model insensitive, isospin breaking in the isovector interaction dependent on the pion mass.

An important finding is that the isospin breaking is small in the isovector amplitude. This is consistent with the finding of Meissner et al. [6], but it is in violent disagreement with the important violation reported by Matsinos [23], which depends only on isospin breaking in the isovector amplitude, although the author does not explicitly state so. The Matsinos results have also been shown to be grossly at variance with the empirical scattering lengths deduced from pionic hydrogen and deuterium [3].

Although therefore the chiral EFT constants $f_{1,2}$ can be rather well understood from physical effects, this is not the whole story. Our terms include the main part, but not all, of the isospin breaking mechanisms. In addition to the effects discussed here, the effective field theory generates generic isospin breaking beyond our considerations. These additional terms appear in charge exchange and $\pi^0 N$ elastic scattering. They are conceptually important non-trivial chiral predictions. Numerically, such contributions are of the order of 1% and of similar magnitude as the terms we consider. Obviously a quantitative test of the predictions of EFT requires that that these intrinsic terms be

reliably separated from those which are generated by the general mechanisms discussed here. A meaningful analysis will require both approaches.

References

- [1] Y. Tomozawa, *Nuovo Cim.* **46A**, 707 (1966).
- [2] S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966).
- [3] T. E. O. Ericson, B. Loiseau, and A. W. Thomas, *Phys. Rev.* **C66**, 014005 (2002).
- [4] D. Gotta *et al.*, *Physica Scripta*, T **104**, 94 (2003), hep-ex/0305012.
- [5] H.-Ch. Schröder *et al.*, *Eur. Phys. J. C* **21**, 473 (2001).
- [6] N. Fettes and U.-G. Meißner, *Phys. Rev. C* **63** 045201 (2001); (hep-ph/0008181).
- [7] N. Fettes and Ulf-G. Meißner, *Nucl.Phys.* **AA679**, 629 (2001).
- [8] J. Gasser, *et al.*, *Eur. Phys. J. C* **26**, 13 (2002).
- [9] T. E. O. Ericson, B. Loiseau, and S. Wycech, *Phys. Lett. B* **594** (2004) 76.
- [10] T. E. O. Ericson and A. Ivanov, to appear.
- [11] S. Deser, *et al.*, *Phys. Rev.* **96**, 774 (1954).
- [12] T. L. Trueman, *Nucl. Phys.* **26**, 57 (1961).
- [13] N. Fettes, *et al.*, *Phys. Lett. B* **451**, 233 (1999).
- [14] T. E. O. Ericson and L. Tauscher, *Phys. Lett.* **B112** 425 (1982).
- [15] N. M. Kroll and M. A. Ruderman, *Phys. Rev.* **93**, 233 (1954).
- [16] J. Spuller *et al.*, *Phys. Lett. B* **67**, 479 (1977).
- [17] T. E. O. Ericson and W. Weise, in *Pions and Nuclei*, Clarendon Press, Oxford, 1988.
- [18] M. M. Nagels *et al.*, *Nucl. Phys.* **B147**, 189 (1979).
- [19] V. E. Lyubovitskij, *et al.*, *Phys. Lett. B* **520**, 204 (2001).
- [20] G. F. Brown and W. Weise, *Phys. Rep.* **22**, 279 (1975).
- [21] J. Gasser and H. Leutwyler, *Nucl. Phys.* **B94** (1975) 269.
- [22] V. E. Lyubovitskij, *et al.*, *Phys. Rev. C* **65** (2002).
- [23] E. Matsinos, *Phys. Rev. Phys. Rev. C* **56** (1997) 3014.