

Recent theoretical studies on hadronic atoms

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A report contains a review of recent theoretical investigations on kaonic atoms carried out at Stefan Meyer Institute of the Austrian Academy of Sciences. We discuss (1) a phenomenological quantum field theoretic model for $\bar{K}N$ interactions at threshold of the reactions $K^-p \rightarrow K^-p$, $K^-n \rightarrow K^-n$ and $K^-d \rightarrow K^-d$ and (2) the energy level displacements of the ground and nP excited states of kaonic hydrogen, (3) the contribution of the $\sigma_{KN}^{I=1}(0)$ -term to the shift of the energy level of the ground state of kaonic hydrogen, (4) the isospin-breaking and dispersive corrections to the energy level displacement of the ground state of kaonic hydrogen, (5) the radiative transitions $nP \rightarrow 1S + \gamma$, induced by strong low-energy interactions and enhanced by the Coulomb interaction, in kaonic hydrogen and kaonic deuterium, (6) perspectives and (7) comments on the approach, where we adduce our recent results obtained after the Workshop EXA05.

1 Kaonic hydrogen

1.1 Experimental data on the energy level displacement of the ground state of kaonic hydrogen

The contemporary experimental and theoretical status of kaonic atoms has been recently outlined by Jürg Gasser [1]. The most recent experimental value on the energy level displacement of the ground state of kaonic hydrogen

$$-\epsilon_{1s}^{(\text{exp})} + i \frac{\Gamma_{1s}^{(\text{exp})}}{2} = (-194 \pm 41) + i(125 \pm 59) \text{ eV}, \quad (1)$$

obtained by the DEAR Collaboration [2], by a factor ~ 2 smaller than the experimental value measured by the KEK Collaboration [3]

$$-\epsilon_{1s}^{(\text{exp})} + i \frac{\Gamma_{1s}^{(\text{exp})}}{2} = (-323 \pm 64) + i(204 \pm 115) \text{ eV}. \quad (2)$$

For the theoretical analysis of the experimental data on the energy level displacement of the ground state of kaonic hydrogen, obtained by the DEAR Collaboration Eq.(1), we have proposed a phenomenological quantum field theoretic model of strong low-energy $\bar{K}N$ interactions at threshold [4].

1.2 Phenomenological quantum field theoretic model for $\bar{K}N$ interactions at threshold

In Ref.[4] (see also [5]) we have proposed a phenomenological quantum field theoretic model for strong low-energy $\bar{K}N$ interactions at threshold and calculated the complex S-wave scattering lengths \tilde{a}_0^0 and \tilde{a}_0^1 of $\bar{K}N$ scattering with isospin $I = 0$ and $I = 1$:

$$\begin{aligned} \tilde{a}_0^0 &= (-1.221 \pm 0.072) + i(0.537 \pm 0.064) \text{ fm}, \\ \tilde{a}_0^1 &= (+0.258 \pm 0.024) + i(0.001 \pm 0.000) \text{ fm}. \end{aligned} \quad (3)$$

The S-wave amplitude of K^-p scattering at threshold is equal to [4]:

$$f_0^{K^-p}(0) = \frac{\tilde{a}_0^0 + \tilde{a}_0^1}{2} = (-0.482 \pm 0.034) + i(0.269 \pm 0.032) \text{ fm}. \quad (4)$$

The theoretical value for the energy level displacement of the ground state of kaonic hydrogen is defined by the DGBTT formula (the Deser–Goldberger–Baumann–Thirring–Trueman formula [6]) (see [4]):

$$-\epsilon_{1s}^{(0)} + i \frac{\Gamma_{1s}^{(0)}}{2} = 412 f_0^{K^-p}(0) = (-203 \pm 15) + i(113 \pm 14) \text{ eV}. \quad (5)$$

The theoretical values of the energy level shift and width Eq.(4) agree well with the experimental data by the DEAR Collaboration [2] Eq.(1) and only qualitatively with the experimental data by the KEK Collaboration [3].

The theoretical ground for our model of low-energy $\bar{K}N$ interactions is a phenomenological Effective Field Theory with Effective Chiral Lagrangians and ChPT [8, 9].

In our model of strong low-energy $\bar{K}N$ interactions near threshold, proposed in [4, 5], the imaginary part of the S-wave amplitude of K^-p scattering is defined by the contributions of the strange baryon resonance $\Lambda(1405)$ and the baryon background with quantum numbers of resonances $\Lambda(1800)$ and $\Sigma(1750)$ ¹. According to Gell–Mann’s $SU(3)$ classification of hadrons, the $\Lambda(1405)$ resonance is an $SU(3)$ singlet, whereas the resonances $\Lambda(1800)$ and $\Sigma(1750)$ are components of an $SU(3)$ octet [7]. This allows to describe the

¹See Chapter 4.

experimental data on the cross sections $\sigma_{Y\pi}$ for the inelastic reactions $K^-p \rightarrow Y\pi$, where $Y\pi = \Sigma^-\pi^+, \Sigma^+\pi^-, \Sigma^0\pi^0$ and $\Lambda^0\pi^0$, at threshold of K^-p scattering [11]

$$\begin{aligned} \gamma &= \frac{\sigma_{\Sigma^-\pi^+}}{\sigma_{\Sigma^+\pi^-}} = 2.360 \pm 0.040 \quad , \quad R_c = \frac{\sigma_{\Sigma^-\pi^+} + \sigma_{\Sigma^+\pi^-}}{\sum_{Y\pi} \sigma_{Y\pi}} = 0.664 \pm 0.011, \\ R_n &= \frac{\sigma_{\Lambda^0\pi^0}}{\sigma_{\Sigma^0\pi^0} + \sigma_{\Lambda^0\pi^0}} = 0.189 \pm 0.015 \end{aligned} \quad (6)$$

with an accuracy of about 6% [4] and the constraint that the $\Lambda(1800)$ resonance decouples from the K^-p system at low energies. We would like to emphasize that for the description of the experimental data Eq.(6) we have not specified the values of the parameters of resonances. We have satisfied the experimental data by using only the $SU(3)$ properties of the resonances $\Lambda(1405)$, $\Lambda(1800)$ and $\Sigma(1750)$ ².

Due to the optical theorem and using the experimental data Eq.(6) the imaginary part $\text{Im} f_0^{K^-p}(0)$ of the S-wave amplitude of K^-p scattering can be expressed in terms of the parameters γ and R_c and the S-wave amplitude $f(K^-p \rightarrow \Sigma^-\pi^+)$ of the inelastic channel $K^-p \rightarrow \Sigma^-\pi^+$ [4]: $\text{Im} f_0^{K^-p}(0) = (1 + 1/\gamma)(1/R_c)|f(K^-p \rightarrow \Sigma^-\pi^+)|^2 k_{\Sigma^-\pi^+} = (0.269 \pm 0.032)$ fm, where $k_{\Sigma^-\pi^+} = 173.85$ MeV is a relative momentum of the $\Sigma^-\pi^+$ pair at threshold of K^-p scattering.

In our model for the calculation of the amplitude $f(K^-p \rightarrow \Sigma^-\pi^+)$ one needs two input parameters g_1 and g_2 , the coupling constants of the interactions $\Lambda(1405)$ and $\Sigma(1750)$ with $B = (N, \Sigma, \Lambda^0, \Xi)$ and $P = (\pi, K, \bar{K}, \eta(550))$, the octets of low-lying baryons and pseudoscalar mesons. The coupling constants $g_1 = 0.907$ and $g_2 = 1.123$ have been calculated in [4], using the recommended values for masses and widths of the resonances $\Lambda(1405)$ and $\Sigma(1750)$ and physical masses of baryons and pseudoscalar mesons [7].

The real part $\text{Re} f_0^{K^-p}(0)$ of the S-wave amplitude of K^-p scattering at threshold, i.e. the S-wave scattering length $\mathcal{R}e f_0^{K^-p}(0) = a_0^{K^-p}$ of K^-p scattering, $\mathcal{R}e f_0^{K^-p}(0) = \mathcal{R}e f_0^{K^-p}(0)_R + \mathcal{R}e \tilde{f}_0^{K^-p}(0)$, is defined by the contribution (i) of the strange baryon resonances in the s-channel of low-energy elastic K^-p scattering, (ii) of the exotic four-quark (or $K\bar{K}$ molecules) scalar states $a_0(980)$ and $f_0(980)$ in the t-channel of low-energy elastic K^-p scattering and (iii) of hadrons with non-exotic quark structures, i.e. $q\bar{q}$ for mesons and qqq for baryons [7], where $q = u, d$ or s quarks. The contribution of the strange resonances $\Lambda(1405)$ and $\Sigma(1750)$, which we denote as $\mathcal{R}e f_0^{K^-p}(0)_R = (-0.154 \pm 0.009)$ fm, explains 32% of the mean value of the S-wave scattering length of K^-p scattering [4]: $\mathcal{R}e f_0^{K^-p}(0) = (-0.482 \pm 0.034)$ fm. The contribution of the exotic scalar mesons $a_0(980)$ and $f_0(980)$ and non-exotic hadrons, calculated to leading order in chiral expansion, is equal to [4]

$$\mathcal{R}e \tilde{f}_0^{K^-p}(0) = 0.398 - 0.614 \xi \text{ fm}, \quad (7)$$

where the first term is defined by strong low-energy interactions of non-exotic hadrons, whereas the second one is caused by the exotic scalar mesons $a_0(980)$ and $f_0(980)$ coupled to the K^- -meson and the proton. The parameter ξ is a low-energy constant (LEC), defining low-energy $a_0(980)NN$ and $f_0(980)NN$ interactions. Using the hypothesis of

²See Chapter 4.

quark–hadron duality, formulated by Shifman *et al.* within non-perturbative QCD in terms of QCD sum rules [12], and the Effective quark model with chiral $U(3) \times U(3)$ symmetry [13], we have calculated the parameter ξ [4]: $\xi = 1.2 \pm 0.1$.

1.3 Energy level displacement of the excited nP states of kaonic hydrogen

The application of our model of strong low-energy $\bar{K}N$ interactions at threshold [4, 5] to the description of K^-p scattering in the P-wave state has allowed to calculate the energy level displacement of the excited nP states of kaonic hydrogen [14]. For the $2p$ excited state we have obtained: $\epsilon_{2p} = -0.6$ meV and $\Gamma_{2p} = 2$ meV.

Faifman and Men'shikov have presented the calculated yields for the K -series of X -rays for kaonic hydrogen in dependence of the hydrogen density [15]. They have shown that the use of the theoretical value $\Gamma_{2p} = 2$ meV of the width of the $2p$ state of kaonic hydrogen, calculated in our model, leads to good agreement with the experimental data, measured for the K_α -line by the KEK Collaboration [3].

1.4 Radiative transitions $nP \rightarrow 1S + \gamma$ in kaonic hydrogen and kaonic deuterium, caused by strong low-energy interactions

The analysis of the transitions $(K^-p)_{np} \rightarrow (K^-p)_{1s} + \gamma$, induced by strong low-energy interactions and enhanced by the Coulomb interaction of the K^-p pair in the initial and final state of the amplitude of the kaon–proton *Bremsstrahlung* $K^- + p \rightarrow K^- + p + \gamma$, has led to the transition rate [16]:

$$\Gamma((K^-p)_{2p} \rightarrow (K^-p)_{1s} \gamma) = 4.3 \times 10^4 \left(\epsilon_{1s}^2 + \frac{1}{4} \Gamma_{1s}^2 \right) \text{sec}^{-1}. \quad (8)$$

For kaonic deuterium we have obtained the following transition rate [16]

$$\Gamma((K^-d)_{2p} \rightarrow (K^-d)_{1s} \gamma) = 3.6 \times 10^4 \left(\epsilon_{1s}^2 + \frac{1}{4} \Gamma_{1s}^2 \right) \text{sec}^{-1}. \quad (9)$$

Measurements of the energy level displacements of the ground states for kaonic atoms at the eV level would demand to take into account the transition rates $(K^-p)_{2p} \rightarrow (K^-p)_{1s} + \gamma$ and $(K^-d)_{2p} \rightarrow (K^-d)_{1s} + \gamma$, given by Eqs.(8) and (9), for the theoretical description of the experimental data on the X -ray spectra and yields.

1.5 Isospin-breaking and dispersive corrections to the energy level displacement of the ground state of kaonic hydrogen

In [17] we have applied our model of strong low-energy $\bar{K}N$ interactions at threshold to the analysis of isospin-breaking corrections to the S-wave amplitude of K^-p scattering [17], caused by the QCD isospin-breaking interaction [18]: $\mathcal{L}_{\text{QCD}}^{\Delta I=1}(x) = \frac{1}{2} (m_d - m_u) [\bar{u}(x)u(x) - \bar{d}(x)d(x)]$. We have shown that these corrections are of order $O(\alpha)$ and make up about 0.04%. Our results agree well with the estimates, obtained by Meißner *et al.* [19] within a

systematic analysis of isospin–breaking corrections in the framework of a non–relativistic Effective Field Theory based on ChPT by Gasser and Leutwyler [8].

For the analysis of the energy level displacements of the $n\ell$ excited states of kaonic hydrogen we have derived a formula relating the energy level displacement to the amplitude of K^-p scattering for arbitrary relative momenta of the K^-p pair, weighted with the wave functions of kaonic hydrogen in the $n\ell$ state [4, 5, 14]. This has allowed to calculate the contribution to the energy level displacement of the ground state of kaonic hydrogen of the intermediate \bar{K}^0n state on–mass shell [17]: $\delta_S^{Disp} = (4.8 \pm 0.4)\%$ and $\delta_W^{Disp} = (8.0 \pm 1.0)\%$, where δ_S^{Disp} and δ_W^{Disp} are corrections to the shift and width, respectively. Such dispersive corrections have not been calculated before in literature.

1.6 Energy level shift of the ground state of kaonic hydrogen and the $\sigma_{KN}^{I=1}(0)$ –term

In our approach the S–wave amplitude of K^-p scattering is calculated to leading order in chiral expansion [4]. This allows to take into account next–to–leading corrections in chiral expansions such as the $\sigma_{KN}^{(I=1)}(0)$ –term, defined by [20]–[24]

$$\sigma_{KN}^{(I=1)}(0) = \frac{m_u + m_s}{4m_N} \langle p | \bar{u}(0)u(0) + \bar{s}(0)s(0) | p \rangle, \quad (10)$$

where m_u and m_s are masses and $u(0)$ and $s(0)$ are operators of the interpolating fields of u and s current quarks.

The contribution of the $\sigma_{KN}^{(I=1)}(0)$ –term to the energy level shift is equal to [14, 17]

$$\delta\epsilon_{1s}^{(\sigma)} = \frac{\alpha^3 \mu^3}{2\pi m_K F_K^2} \left[\sigma_{KN}^{(I=1)}(0) - \frac{m_K^2}{4m_N} i \int d^4x \langle p | \text{T}(J_{50}^{4+i5}(x) J_{50}^{4-i5}(0)) | p \rangle \right], \quad (11)$$

where $J_{50}^{4\pm i5}(x)$ are time–components of the axial hadronic currents $J_{5\mu}^{4\pm i5}(x)$, changing strangeness $|\Delta S| = 1$, $F_K = 113$ MeV is the PCAC constant of the K –meson [7].

Taking into account the contribution $\delta\epsilon_{1s}^{(\sigma)}$, the total shift of the energy level of the ground state of kaonic hydrogen is equal to [14, 17]

$$\epsilon_{1s}^{(\text{th})} = (213 \pm 15) + \frac{\alpha^3 \mu^3}{2\pi m_K F_K^2} \left[\sigma_{KN}^{(I=1)}(0) - \frac{m_K^2}{4m_N} i \int d^4x \langle p | \text{T}(J_{50}^{4+i5}(x) J_{50}^{4-i5}(0)) | p \rangle \right]. \quad (12)$$

The theoretical estimates of the value of $\sigma_{KN}^{(I=1)}(0)$, carried out within Effective Field Theory approach based on ChPT with a dimensional regularization of divergent integrals, are converged around the number $\sigma_{KN}^{(I=1)}(0) = (200 \pm 50)$ MeV [22, 23]. Hence, the contribution of $\sigma_{KN}^{(I=1)}(0)$ to the energy level shift amounts to (67 ± 17) eV. This gives

$$\epsilon_{1s}^{(\text{th})} = (280 \pm 23) - \frac{\alpha^3 \mu^3 m_K}{8\pi F_K^2 m_N} i \int d^4x \langle p | \text{T}(J_{50}^{4+i5}(x) J_{50}^{4-i5}(0)) | p \rangle. \quad (13)$$

The theoretical analysis of the second term in Eq.(13) is required for the correct understanding of the contribution of the $\sigma_{KN}^{I=1}(0)$ –term to the shift of the energy level of the ground state of kaonic hydrogen.

Of course, one can solve the inverse problem. Indeed, carrying out a theoretical calculation of the term $i \int d^4x \langle p | T(J_{50}^{4+i5}(x) J_{50}^{4-i5}(0)) | p \rangle$ in Eq.(12) and using the experimental data on the shift of the energy level of the ground state of kaonic hydrogen, measured by the DEAR Collaboration, one can extract the value of the $\sigma_{KN}^{(I=1)}(0)$ -term. Such a value of the $\sigma_{KN}^{(I=1)}(0)$ -term should be compared with the theoretical predictions, obtained within the Effective Field Theory approach based on ChPT.

2 Kaonic deuterium

The energy level displacement of the ground state of kaonic deuterium is given by the DGBTT formula [6] (see also [5]):

$$-\epsilon_{1s} + i \frac{\Gamma_{1s}}{2} = 602 f_0^{K^-d}(0), \quad (14)$$

where $f_0^{K^-d}(0)$ is the S-wave amplitude of K^-d scattering at threshold.

In our model of strong low-energy KN interactions [5] we have shown that the S-wave scattering length $\mathcal{R}e f_0^{K^-d}(0) = a_0^{K^-d}$ of K^-d scattering is fully defined by the Ericson-Weise scattering length for K^-d scattering in the S-wave state [25], which we denote as $(a_0^{K^-d})_{EW}$, i.e. $a_0^{K^-d} \simeq (a_0^{K^-d})_{EW}$. The S-wave scattering length $(a_0^{K^-d})_{EW}$ is equal to [5]

$$\begin{aligned} (a_0^{K^-d})_{EW} &= \frac{1 + m_K/m_N}{1 + m_K/m_d} (a_0^{K^-p} + a_0^{K^-n}) \\ &+ \frac{1}{4} \left(1 + \frac{m_K}{m_d}\right)^{-1} \left(1 + \frac{m_K}{m_N}\right)^2 ((a_0^1)^2 + 4a_0^0 a_0^1 - (a_0^0)^2) \left\langle \frac{1}{r_{12}} \right\rangle, \end{aligned} \quad (15)$$

where r_{12} is a distance between two scatterers n and p [25]. In our approach $\langle 1/r_{12} \rangle$ is defined by [5]

$$\left\langle \frac{1}{r_{12}} \right\rangle = \int d^3x \Psi_d^*(\vec{r}) \frac{e^{-m_K r}}{r} \Psi_d(\vec{r}) = 2\gamma_d E_1 \left(\frac{m_N}{m_K + 2\gamma_d} \right) = 0.29 m_\pi, \quad (16)$$

where $\Psi_d(\vec{r})$ is the wave function of the deuteron in the ground state.

We would like to remind that Ericson and Weise did not investigate the S-wave scattering length of K^-d scattering. They analysed only π^-d scattering. However, since the structure of the term Eq.(15) is very similar to that of π^-d scattering we call such a term as the Ericson-Weise scattering length $(a_0^{K^-d})_{EW}$. At the quantum field theoretic level the Ericson-Weise scattering length has been derived in [5].

The imaginary part $\mathcal{I}m f_0^{K^-d}(0)$ of the S-wave amplitude of K^-d scattering at threshold we have calculated in terms of the contributions of the two-body reactions $K^-d \rightarrow NY$, where $NY = n\Lambda^0$, $n\Sigma^0$ and $p\Sigma^-$, and the experimental data on the two-body production rates [26]. Our theoretical predictions for the ratios of the two-body production rates [5]: $R(\Lambda^0/\Sigma^0) = 1.0 \pm 0.3$, $R(\Sigma^0/\Sigma^-) = 0.8 \pm 0.2$, $R(\Lambda^0/\Sigma^-) = 0.8 \pm 0.2$ agree well with the experimental data [26]. The theoretical values of the partial widths of two-body inelastic decay channels $K^-d \rightarrow NY$, obtained in our model, are equal to [5]

$$\Gamma_{1s}^{(n\Lambda^0)} = (2.2 \pm 0.5) \text{ eV}, \Gamma_{1s}^{(n\Sigma^0)} = (2.4 \pm 0.5) \text{ eV}, \Gamma_{1s}^{(p\Sigma^-)} = (3.0 \pm 0.6) \text{ eV}. \quad (17)$$

Using the experimental value of the total production rate $R = (1.229 \pm 0.090)\%$ [26] and our theoretical predictions for the widths of the two-body decay channels (17), we can estimate the expected value of the total width of the ground state of kaonic deuterium

$$\Gamma_{1s} = \frac{\Gamma_{1s}^{(n\Lambda^0)} + \Gamma_{1s}^{(n\Sigma^0)} + \Gamma_{1s}^{(p\Sigma^-)}}{(1.229 \pm 0.090) \times 10^{-2}} = (630 \pm 100) \text{ eV}. \quad (18)$$

The S-wave amplitude of K^-d scattering at threshold is equal to [5]:

$$f_0^{K^-d}(0) = (-0.540 \pm 0.095) + i(0.521 \pm 0.075) \text{ fm} \quad (19)$$

Our prediction for the energy level displacement of the ground state of kaonic deuterium [5]

$$-\epsilon_{1s} + i \frac{\Gamma_{1s}}{2} = 602 f_0^{K^-d}(0) = (-325 \pm 60) + i(315 \pm 50) \text{ eV}. \quad (20)$$

can be used for the planning experiments by the DEAR/SIDDHARTA Collaborations at Frascati [27].

3 Perspectives

Within our approach to the description of strong low-energy $\bar{K}N$ interactions one can solve the following problems :

- The extraction of the value of the $\sigma_{\bar{K}N}^{I=1}$ -term from the experimental data by the DEAR Collaboration on the energy level displacement of kaonic hydrogen
- The calculation of the energy level displacement of the ground state of kaonic deuterium, caused by three-body inelastic channels $K^-d \rightarrow NY\pi$
- The analysis of the contribution of the reaction $K^-d \rightarrow \bar{K}^0 nn \rightarrow K^-d$ to the energy level displacement of the ground state of kaonic deuterium
- The analysis of the contribution of the $\sigma_{\bar{K}N}^{I=1}$ -term to the shift of the energy level of the ground state of kaonic deuterium
- The calculation of the energy level displacement of the excited nP states of kaonic deuterium
- The calculation of the energy level displacement of the ground state and nP excited states of kaonic ${}^3\text{He}$ and ${}^4\text{He}$
- The calculation of the radiative transition rates $nP \rightarrow 1S + \gamma$ in kaonic ${}^3\text{He}$ and ${}^4\text{He}$, induced by strong low-energy interactions and enhanced by the Coulomb interaction in the $K^-{}^3\text{He}$ and $K^-{}^4\text{He}$ pairs
- The analysis of the deeply bound states (K^-pp) and (K^-NNN).

4 Comments on the approach

4.1 Kaonic hydrogen revisited

In our recent analysis of the S-wave scattering lengths $a_0^{K^-0} = (a_0^0 + a_0^1)/2$ and $a_0^{K^-n} = a_0^1$ of K^-p and K^-n scattering, carried out in [28], we have shown that the S-wave scattering lengths $a_0^{K^-0}$ and $a_0^{K^-n}$, calculated to leading order in chiral expansion, satisfy the low-energy theorem

$$a_0^{K^-p} + a_0^{K^-n} = \frac{1}{2} (a_0^0 + 3a_0^1) = 0. \quad (21)$$

We have derived this low-energy theorem relating the S-wave scattering lengths of K^-N scattering to the S-wave scattering lengths of π^-N scattering. As has been shown by Weinberg [29], in the chiral limit the S-wave scattering lengths $a_0^{\pi^-p} = (2a_0^{1/2} + a_0^{3/2})/3$ and $a_0^{\pi^-n} = a_0^{3/2}$ of π^-N elastic scattering³ obey the constraint $a_0^{\pi^-p} + a_0^{\pi^-n} = 2(a_0^{1/2} + 2a_0^{3/2})/3 = 2b_0^0 = 0$, which is caused by Adler's consistency condition⁴. We have shown [28] that due to isospin invariance of strong low-energy interactions in the chiral limit the sum of the S-wave scattering lengths $a_0^{K^-p} + a_0^{K^-n}$ is equal to $a_0^{K^-p} + a_0^{K^-n} = -\sqrt{6}b_0^0 = 0$. The same result can be obtained by using U -spin invariance of strong low-energy interactions in the chiral limit. According to U -spin classification, the K and π mesons are components of U -spin doublets.

We have found that the $\Sigma(1750)$ resonance does not saturate the low-energy theorem Eq.(21), therefore we have replaced the contribution of the $\Sigma(1750)$ resonance by the phenomenological baryon background with quantum numbers of the $\Sigma(1750)$ resonance [28]. The value of this background we have fixed in terms of the parameter γ Eq.(6) and the contribution of the $\Lambda(1405)$ resonance, which is left unchanged.

For the complex S-wave scattering lengths \tilde{a}_0^0 and \tilde{a}_0^1 of $\bar{K}N$ scattering with isospin $I = 0$ and $I = 1$ we have obtained the numerical values

$$\begin{aligned} \tilde{a}_0^0 &= (-1.50 \pm 0.05) + i(0.66 \pm 0.01) \text{ fm}, \\ \tilde{a}_0^1 &= (+0.50 \pm 0.02) + i(0.04 \pm 0.00) \text{ fm}, \end{aligned} \quad (22)$$

where $\mathcal{R}e \tilde{a}_0^0 = a_0^0 = (-1.50 \pm 0.05) \text{ fm}$ and $\mathcal{R}e \tilde{a}_0^1 = a_0^1 = (+0.50 \pm 0.02) \text{ fm}$.

It is seen that the real parts of the complex scattering lengths a_0^0 and a_0^1 satisfy the low-energy theorem Eq.(21). This makes more believable the investigation of the $\sigma_{KN}^{I=1}(0)$ -term contribution, which gives the next-to-leading order correction in chiral expansion to the shift of the energy level of the ground state of kaonic hydrogen.

The complex S-wave scattering length \tilde{a}_0^0 agrees well with the result obtained by Dalitz and Deloff [30] $\tilde{a}_0^0 = (-1.54 \pm 0.05) + i(0.74 \pm 0.02) \text{ fm}$ for the position of the pole on sheet II of the E -plane $E^* - i\Gamma/2$ with $E^* = 1404.9 \text{ MeV}$ and $\Gamma = 53.1 \text{ MeV}$ [30]. This corresponds to our choice of the parameters of the $\Lambda(1405)$ resonance.

It is interesting to notice that the complex S-wave scattering length \tilde{a}_0^0 , calculated in our model, does not contradict the result obtained by Akaishi and Yamazaki [31] at the assumption that the $\Lambda(1405)$ resonance is the bound K^-p state.

³Here $a_0^{1/2}$ and $a_0^{3/2}$ are the S-wave scattering lengths of πN scattering with isospin $I = 1/2$ and $I = 3/2$.

⁴We remind that b_0^0 is the S-wave scattering length of πN scattering in the t -channel with isospin $I = 0$.

For the energy level displacement of the ground state of kaonic hydrogen, calculated for the S-wave amplitude of K^-p scattering at threshold Eq.(22), we get

$$-\epsilon_{1s}^{(0)} + i \frac{\Gamma_{1s}^{(0)}}{2} = 412 f_0^{K^-p}(0) = 412 \frac{\tilde{a}_0^0 + \tilde{a}_0^1}{2} = (-205 \pm 21) + i(144 \pm 9) \text{ eV}. \quad (23)$$

This result agrees well with the experimental data by the DEAR Collaboration.

In addition we have shown that the cross sections for elastic K^-p scattering and inelastic channels $K^-p \rightarrow \Sigma^- \pi^+$, $K^-p \rightarrow \Sigma^+ \pi^-$, $K^-p \rightarrow \Sigma^0 \pi^0$ and $K^-p \rightarrow \Lambda^0 \pi^0$, calculated in our model for laboratory momenta of the incident K^- meson from domain $70 \text{ MeV}/c \leq p_{lab} \leq 150 \text{ MeV}/c$, satisfy the available experimental data within two standard deviations. These results agree well with the theoretical analysis of low-energy $\bar{K}N$ interactions, carried out in [32] within the $SU(3)$ effective chiral Lagrangian approach and relativistic coupled channels technique.

4.2 Kaonic deuterium revisited

Our theoretical predictions for the S-wave amplitude of K^-d scattering and the energy level displacement of the ground state of kaonic deuterium, given in [5], are left practically unchanged.

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