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Theoretical aspects of asteroseismology

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1. Introduction

For a few decades, a large international community is working at the development of stellar seismology, intending to open a new window on the structure and evolution of stars, with the necessary parallel development of new observational techniques, modeling tools and methods for diagnostics and interpretation. This new window is associated with new observables, characterizing the oscillations (frequencies, amplitudes, line profiles,...) which are sensitive to the stellar interior and thus very complementary with surface observables, associated to astrometry, spectropolarimetry or interferometry.

Strong efforts have been made on both the observational side, to bring observational data suitable for seismic interpretation, and on the theoretical side, to refine stellar evolution models and bring them at the level of precision required by seismology sensitivity. However, ultimately, stellar seismology resides in our practical ability to confront observations and models and reach a seismic diagnostic.

I will focus here on this interpretation aspect of stellar seismology, taking examples selected in the recent literature to illustrate the present problematics, the difficulties and the present approach of the problem. I hope to help this way the participants of this session, especially the youngest and the new comers to this field, to get some references and keys to appreciate the different works which will be presented in this session and identify how they do participate to the development of this field.

I will first briefly sketch what the goals of stellar seismology are. Then I will introduce some of the most commonly used tools and come to a few hopefully illustrative tentative applications selected in the literature.

Finally, I will propose a point of view on the present state of the art and prospects.

2. About goals of stellar seismology

Goals of stellar seismology are to a great extent strongly related to the open questions of stellar structure and evolution.

The mainlines of stellar structure and evolution have been understood by confrontation of observables coming from the surface of stars and theoretical modeling calling to a wide panel of various fields of physics. Our understanding of stellar evolution and our capacity to describe it precisely is thus suffering large uncertainties, due to the fact that a star is a complex object, involving a large amount of physical processes still poorly understood.

Considering the upper part of the main sequence for instance, which is characterized by the existence of a convective core, one of the most debated open question is how the edge of the convective mixed core is or not extended by the so-called overshooting process. This point alone, by the change induced in the amount of hydrogen available for central nuclear reactions, is responsible for an uncertainty which can reach 30 to 50% in the age estimate of all stars with mass higher than $\sim 1.1\mathrm{M}_\odot$ (for solar composition).

For the lower part of the main sequence ($M \leq 1.4M\odot$), characterized by an extended outer convective zone below the surface, one of the most prominent open questions deals with the efficiency of the heat transport in upper convective regions. There, density is so low and convective heat transport so inefficient that the uncertainty in determining the temperature gradient becomes important and hampers severely our description of these stars.

In spite of several tentative refinements, these two processes are widely considered at the moment in the modeling by simple one-parameter crude descriptions.

Beyond these two points, segregation of the different chemical species induced by gravitational and radiative forces are considered as a non negligible factor in several classes of objects, with a prominent manifestation in the surface anomalies observed in A stars. Impressive efforts have been made to implement these aspects (e.g. Michaud 2004), but they remain very heavy in computing time and their use very restricted.

Mass loss, meridional circulation and turbulence, their influence on angular momentum evolution, their interaction with previously quoted diffusion of chemical elements constitute another active front for research in this field (e.g. Talon 2004), but are not yet integrated to the standard model.

On top of this, one can consider the effect of magnetic field and its potential interaction with the previous mechanisms (Alecian 2004, Mathis & Zahn 2005).

In the pre-Main Sequence phase, it is still an open question to know how the angular momentum evolves and which influence it has on the structure at the beginning of the Main Sequence phase.

At the other end of stellar evolution, the chemical composition profile of the strongly stratified structure of white dwarfs is holding the signature of e.g. $C^{12} - O^{16}$ poorly know reaction rate or mixing processes at work during the red giant phase.

Oscillations have been observed in stars representative of approximately all mass ranges and evolution stages, from the PMS stage, to the white dwarf cooling sequence, including Main Sequence, horizontal branch and red giant phases. New classes of stellar pulsators are still regularly discovered, and pulsation looks now more like the rule rather than the exception. This definitely suggests that seismology is a promising tool to improve our understanding of the various physical processes at work in stars.

3. About tools of stellar seismology

Stars are generally seen pulsating on a more or less extended set of (low degree l) eigenmodes, as described previously by Gerald Handler (these proceedings). These eigenmodes are standing waves established inside the stars by essentially two types of propagation waves: pressure waves, for which the restoring force is dominantly the pressure gradient, and gravity waves, for which the restoring force is essentially the buoyancy.

The associated eigenfunctions can generally be decomposed in a product of a function f(r) describing the radial dependence and the angular dependence expressed by a spherical harmonic (see e.g. Unno et al. 1989). Each eigenmode can thus be characterized by three integers: l and m, the degree and the azimuthal order of the spherical harmonic, and the radial order n, a count of the nodes of function f(r).

The basic idea of stellar seismology is to use these new observables (frequencies, amplitudes, profiles,phase, ...) which are sensitive to the interior structure in a differential way, to help constraining the stellar structure and its evolution.

3.1 Sensitivity to structure - seismic indexes

If we consider, as an example, the oscillation eigenspectrum of a typical Main Sequence object of intermediate mass, we first notice in the upper frequency domain, a high regularity in the distribution of eigenfrequencies associated with p-modes.

This point is explained by the fact that high order p-modes eigenfrequencies follow, to a low order approximation, an asymptotic distribution showing regular spacings in frequency: $\nu_{n,l} \simeq (n + l/2 + \epsilon)\Delta\nu_0 - [l(l+1) + \delta]\frac{A}{n+l/2+\epsilon} + ...$ with $\Delta\nu_0$ commented hereafter and ϵ, δ and A constants involving integrals over the stellar structure (see e.g. Tassoul 1980, Christensen-Dalsgaard 1984)

To this precision, the upper part of the eigenspectrum can thus be characterized by a few parameters. One of them, known as the large separation $\Delta\nu_0 =< \Delta\nu_{n,l} >=< \nu_{n,l} - \nu_{n-1,l} >$ is inversely proportional to the time spent by a wave to travel from the center to the surface ($\Delta\nu_0 \sim [2\int_0^R dr/c]^{-1}$). It is thus roughly proportional to the square root of the mean density.

A second parameter, the small separation $\delta\nu_l =< \delta\nu_{n,l,l+2} >=< \nu_{n,l} - \nu_{n-1,l+2} >$, is sensitive to the gradient of the sound speed, with a strong ponderation toward the interior; it is thus known to be sensitive to the evolution stage of the star, via the change in the core structure.

With these two quantities, it has been proposed by Christensen-Dalsgaard (1988) to build an asteroseismic diagramme, which was shown to be mostly discriminant in the lower mass domain, on the opposite to the classical HR diagramme to which it thus could constitute a helpful complement.

Besides this, other similar parameters have been proposed, depending on which modes are finally observed, e.g. $d_{0,1} = (\nu_{n+1,l=0} + \nu_{n,l=0})/2 - \nu_{n,l=1}, ...$

This asymptotic behavior, also reveals, that to this order, the information carried by eigenfrequencies in this high frequency (high radial order n) domain is highly redundant. If we want to know more about the internal structure, we have to go beyond this low order behavior.

When looking closer, we see that the large separation, as the small one do depend on frequency; they show departure from the asymptotic behavior.

The second order difference $\delta^2_{n,l}\nu = \nu_{n+1,l} - 2\nu_{n,l} + \nu_{n-1,l}$ has been proposed to follow this departure from the asymptotic behavior in the case of the large separation; it corresponds to a finite difference estimate of the derivative of the large separation. Gough (1990) showed that the departure from the asymptotic revealed by this index is mostly dominated by two superposed pseudo-oscillations: one of them being due to the sharp variation of the Γ_1 adiabatic exponent, and thus of the sound speed, in the H and He partial ionization zones; the other, due to the discontinuity in the second derivative of sound speed at the base of the outer convection zone, where the temperature gradient goes from radiative to adiabatic.

In each case, the pseudo-period is related to the acoustic depth of the discontinuity, the amplitude to the sharpness and strength of this discontinuity.

One of the difficulties in interpreting absolute frequencies or even seismic indexes like the large and small separations is that they remain sensitive to the surface layers which are poorly known.

Recently, a new seismic index has been proposed by Roxburgh and Vorontsov (2003); it is in fact a combination of indexes $r_{n,0,2} = \delta\nu_{n,0,2}/\Delta\nu_{n,1}$, or $\delta\nu_{n,1,3}/\Delta\nu_{n,0}$, which have been shown to be less dependent on surface layers.

In the low frequency domain corresponding to high order g-modes, one can see that an asymptotic regime also exists, with regular spacings in period this time.

As described by Tassoul (1980): $p_{n,l} = 1/\nu_{n,l} = (n + \epsilon')2\pi^2/\Im$, with ϵ' a constant depending on the structure of the star and eventually on l when a convective core exists, $\Im$ representing the integral of the Brunt-Väisälä frequency over the propagation cavity of these modes.

Thus, seismic indexes can be derived in this domain of the spectrum too. This has not been used much until now, since observations of high order g-modes have not yet been so common except for white dwarfs, but we will see in the next section a recent example of tentative application of this description to γ Dor objects.

For the intermediate range of frequency, around the fundamental radial mode, one can see that frequencies show strong departure from regular spacing. One can notice the existence of so-called mixed modes, propagating like g-modes in the inner part of the star and as p-modes in the outer part. In this domain, one has to rely on numerical estimates of eigenfrequencies. These modes however, by the diversity of their eigenfunctions, are very promising to bring information on the different regions of the stellar interior.

The direct use of absolute frequencies is not straightforward. Each frequency is generally sensitive to a large domain of the stellar interior, eventually to boundary layers description, rotation... In principle, it is possible to combine frequencies measured for several modes in order to obtain a measurement sensitive to a restricted domain of the stellar interior. This however generally requires a large number of modes and a sufficient knowledge of the eigenfunctions associated with eigenfrequencies, i.e. to have already a close enough model of the observed star. It has been possible for the sun, where inversion of the sound speed and rotation have been obtained (except for the very central part). It has not been applied to stars yet. In the case of stars other than the sun, the most common approach so far makes use of seismic parameters, built with a limited number of eigenfrequencies as commented in the next section.

Modes are also sensitive to rotation inside the star. Multiplets of the same l and n index ($2l+1$ components) are associated with the same eigenfrequency when the star is not rotating. This degeneracy is lifted when the star rotates. For slow rotation, a first order perturbative treatment of the rotation is sufficient, revealing multiplet symmetrically splitted in frequency, the axisymmetric (m=0) modes being unperturbed.

Via the different eigenfunctions associated to them these multiplets are sensitive to rotation in different regions of the star.

For moderate rotation, a perturbative approach to a higher order is still possible. The splitting pattern becomes asymmetric and even m=0 modes see their frequency change.

For fast rotation, a perturbative treatment becomes questionable and another approach is needed.

While eigenfrequencies are characteristics of the equilibrium structure, amplitudes and line-width are determined by energetic aspects, the excitation mechanism and the eventual intrinsic damping time of the modes. Essentially two types of mechanisms are considered to be responsible for the oscillations of known pulsating stars. One is an opacity mechanism (κ-mechanism), where energy is cyclically taken from the radiative flux to feed the oscillation. The second is the one responsible for oscillations in the Sun, where energy is taken from convection eddies to re-excite stochastically the oscillation modes.

Both mechanisms take place near the surface of the star, and their dependence on the physical conditions there, make amplitudes and line-width potential sources of information on these regions, as illustrated hereafter.

4. A few examples of seismic tentative interpretation

4.1 The Beta Cephei ν Eri

The star ν Eri, belongs to the β Cephei pulsators, a class of massive near Main Sequence stars of spectral type B0 to B2, whose oscillations are explained by the κ-mechanism. In agreement with theoretical stability analysis, these objects seem to pulsate in low order p-modes and g-modes. As such, the modeling of their oscillations cannot be described by any asymptotic description and one has to rely on numerical computations.

Tentative seismic interpretations of such objects can be found in a few recent studies. They, however, remain restricted to a low number of modes (3 at maximum). The work by Ausseloos et al. (2004), with 4 modes considered, is on one hand representative of these previous works for the strategy and the results, and at the same time prefiguring a new qualitative step to come.

The most striking point in the previous works is probably to possibility to obtain a solution by fitting the observed absolute frequencies with a high precision, and this for standard models. Of course, the quality of the fit can be partially explained by the 'simplicity' of the structure of these objects, an argument often used to distinguish them from cooler objects in which larger outer convective zones or faster rotation make the modeling more complicated. This is letting aside diffusion processes, however. Having in mind the large set of theoretical inputs, such an agreement could still seem questionable.

Several observational campaigns, both in photometry and spectroscopy have brought 8 detected frequencies and 6 suspected ones, plus an indication of the identification for 4 of these modes. This object has been modeled several times, with 2, 3 and now 4 frequencies.

As a starting point, the authors compute a large grid of standard stellar models, using up-to-date physics (nuclear reaction rates, equation of state, opacity tables), a standard heavy elements mixture, and covering a relevant range of mass, age, Z/H abundances, as well as a range of values for the overshooting parameter to parametrize uncertainty in the treatment of this process. Then, for all these models, they compute the theoretical frequency spectrum.

For ν Eri, the measured $v \sin i$ is below 25 km $\mathrm{s}^{-1}$and since most of these objects are slow to moderate rotators, the authors make the hypothesis that intrinsic rotation is low enough to have no significant effect on frequencies of axisymmetric modes (m=0).

They thus decide to decouple the study, keeping splitted multiplets for study of the rotation and using the 4 identified m=0 modes to compare the observed star and the grid of models with no rotation.

Then, the authors explore the model grid, searching for models which would at the same time satisfy observational error boxes on global parameters (Teff, L), and match the observed frequencies. In addition to this, they search for observed modes to be predicted unstable by non-adiabatic computations for the considered models.

The authors first consider 2 observed frequencies and show that they can find a large set of models of different mass, and different Fe/H value, satisfying all constraints in their grid.

With three observed frequencies, the authors still can find a large set of models satisfying all the observational constraints. At these stage, they thus reproduce what had been obtained in previous works on ν Eri or other β Cephei objects, for which observed frequencies had been explained with standard models, tuning parameters.

Then, considering the fourth frequency, no solution remains, except a very marginal one regarding the value of the overshooting parameter, and in addition in strong contradiction with stability analysis.

The authors thus conclude that they reach a contradiction for standard models, and build 'non-standard' models, where they increase arbitrarily the Fe relative abundance in the initial mixture. This way, they obtain a better (but not fully satisfactory) fit. However, they do not try to go further, stressing that the treatment of diffusion of chemical species, will be required for a more refined description.

In their conclusion, the authors remind that the goal of asteroseismology is the improvement of our current understanding of stellar physics, but present as a prerequirement to first refine the determination of global parameters. However, comparing the parameters they determine for their best solution with the one of previous works, they show that, beyond the number of frequencies considered, this determination is very sensitive to the choice of the observables and to the constraints on the range of values considered for these observables.

4.2 The γ Dor HD 12901

HD 12901 is a γ Dor pulsator, a class known for less than ten years, for which the mechanism responsible for the oscillations is just beginning to be understood. These objects are lying on the Main Sequence, on or just to the right of the red edge of the Cepheid Instability strip. They are pulsating in high order g-modes. Very few tentative interpretations exist for this young class of pulsators and the work on HD 12901, by Moya et al. (2005) although it was more intended to illustrate a method than to present a thorough seismic study, seems interesting because it proposes an approach making use of g-modes asymptotic properties.

This approach is based on the first order asymptotic description proposed by Tassoul (1980). Moya et al., propose to use frequency ratios of observed modes, to determine the order n of the respective modes, as well as to estimate the value of the integral $\Im$ (see above). This first requires to restrict the domain of possible models, considering fundamental parameters (Teff, log g) determined otherwise.

HD 12901 is known to pulsate with 3 frequencies, which is given as the minimum for this method.

As we mentioned already, this work was more intended to be illustrative than conclusive, thus only one value of the overshooting parameter and of the mixing-length parameter were considered, while large uncertainties exist on these parameters, not to mention the numerical description they are associated with. However, the results obtained remain illustrative in our opinion. With such an approach, the authors find a set of models satisfying fundamental parameters determination and frequency ratios and the $\Im$ value estimate. They consider that this approach made it possible to restrict the set of possible models and propose as a further step to search the set of possible models comparing absolute frequencies with observed ones. Doing so, they restrict further the set of possible models but remark that it still contains models with very different ages, log g, etc...

4.3 The ro Ap HD 60435

Rapidly oscillating Ap stars are illustrative of the importance of element diffusion in intermediate mass stars. The surface anomalies are explained by a competition between radiative acceleration pushing upward elements with high opacity (like most metals) and gravity, leading others like helium to sink. To this picture, one has to add rotation-induced mixing, mass loss, etc...,. Among peculiar A stars, some have large magnetic fields (so-called Ap), they show evidence of spots modulated by the magnetic structure.

The instability of the observed modes has been searched in the κ-mechanism associated with helium or hydrogen with specific distribution. The observed modes are low-l high order p-modes.

In the example considered here (Vauclair & Théado 2004), the authors propose to characterize the helium gradient eventually let by diffusion with the pseudo oscillation in the second order difference (see above). As mentioned by the authors, one must distinguish the signature of this helium gradient from the well-known signature of the Γ_1 bump associated with He ionization zone when helium did not sink.

With 17 observed frequencies, HD 60435 is the best opportunity to apply this technique. However, calculating second order differences is very demanding and the number of points obtained (8) is still too low to be conclusive, even in this specific case.

4.4 The solar-like pulsators α Cen A & B

Among the handful of 'solar-like' pulsators for which oscillations have been observed from the ground already, the binary system α Cen A and B deserves a special interest. The orbital elements have been measured as well as angular diameters. A relatively precise trigonometric parallax is available, which allows to derive the masses and radii of both objects. These two objects are interestingly bracketing the sun in mass, for an age probably slightly higher. Photometric and spectroscopic analyses exist, providing estimates of metallicity content, luminosities, effective temperature, etc... In addition to these classical 'non-asteroseismic' constraints for the modeling, oscillations have been detected in both objects, which allow to derive individual frequencies and different seismic indexes introduced hereabove.

Several studies have been published already. We will consider the recent one by Miglio and Montalban (05), as illustrative of the previous ones under several aspects while proposing a slightly new approach.

In this work, the authors consider on one hand parameters for the modeling (M_A,M_B,Z,Y, α_A, α_B, age) and on the other hand, 'observables' to be fitted: L_A, L_B, $Teff_A$, $Teff_B$, R_A, R_B, Z/X_s, and seismic ones: $\Delta\nu$, $\delta\nu$, r_{02},...

As in previous works, the authors propose to search among models parametrized as commented herebefore, the solution which would minimize a χ^2 difference with a given selected set of observables. One originality here is that, instead of intending to start from a complete grid which is very costly in computation time and thus often leads to unjustified restrictions, the authors propose to start from an approximate solution around which derivatives of the solution on different parameters are numerically computed and used in a gradient-expansion algorithm to approach a better solution iteratively.

Thanks to the flexibility of their approach, they give examples of how it can be applied considering different sets of observables. They thus investigate how the choice of observables does impact on the result of the fit. Their results tend to confirm, for instance, that the use of the large separation, potentially sensitive to the poor description of surface effects, might lead to erroneous radii, inconsistent with the measured ones. They show that the use of the $r_{0,2}$ index might be preferable, if it is admitted that the large separations observed differ from the theoretical ones by a significant amount attributed to surface effects.

Their discussion stresses how difficult it is at the moment, when some inconsistency is reached, to determine the relative confidence one can have in a specific seismic index versus another observable, e.g. a radius determination.

The authors also consider different formulations of convective transport in the outer convective zone, different equations of state, microscopic diffusion, overshooting,... but they show that the present error bars on observational data remain too large to tackle these problems.

5. Discussion and perspective

The previous set of examples is incomplete, because of time and space available. Among the most obvious lacks, one could think about white dwarfs and subdwarf B stars (EC14026) which have been the object of a strong activity of seismic interpretation during the past decade (see e.g Metcalfe et al. 2005, Charpinet et al. 2004). One should also consider δ Scuti stars, for which several attempts of seismic interpretation have been published already (see e.g. Suarez et al. 2005, Fox Machado et al. 2005). However, considering all these works, it is possible to determine a few common aspects, which we hope instructive, and which are to some extent illustrated in the previous examples.

In a large majority, not to say all cases, we are still in the approach of model fitting, hopefully leading to some constraints on stellar fundamental parameters, and in some cases on some parametrized physical processes or theoretical options.

At a first look, this approach might look desperate, since its success would suppose that the numerical modeling of the star, of all the physical processes susceptible to have an influence on frequencies, or at least on the selected seismic indexes, are considered and well described...This sounds hopeless but is necessary.

The problem considered is nonlinear under many aspects and it seems a necessary first step to reach a close-enough model to compare seismic observables and get a seismic diagnostic. We need to progress in our understanding of the global problem and find or confirm which parameters or aspects can be neglected or not in a given evolution stage and mass range.

In fact, when applied to a limited number of observed frequencies and/or to a single specific object, we see that a satisfying fit can usually be found, but it looks very much like an ad hoc adjustment of the free parameters.

The most advanced examples confirm that pushing forward this strategy logically leads essentially to reveal the inconsistency of the standard model. At the moment, in the very few cases for which this step has been reached, there is still room to go further by extending the domain of solutions to 'non-standard models', but these 'non-standard' aspects are associated with processes which we suspect to be important, but hardly can pretend to parametrize properly.

As a consequence, it is very likely that further studies, with more detected frequencies, or other objects will, in a first stage, essentially lead to the constatation of incoherences between models, standard or not, and observations.

The next qualitative step will be to recognize these incoherences and analyze them in terms of structural differences. As it appears in the previous examples, it is not straightforward presently to disentangle the diagnostics in terms of physics. To a large extent, it comes from the fact that, in a specific case, even in the best known ones, the conclusion can be substantially changed by an eventual error on one classical observable and this can hardly be totally ruled out. This also comes from the fact that the seismic observables have only been tested in very few practical cases yet, and their eventual sensitivity to e.g. surface effects is still largely unknown.

In this perspective, the most promising point seems to reside in the increase of the number of objects for which such studies can be done. By increasing this number, it should be possible to cross informations and find out coherence in the inconsistencies which would help to disentangle what is specific to one object with a given age, mass or rotation profile, and what is common and can be interpreted as a difference between our modeling and observed objects. This is why it is crucial to increase and diversify the set of objects for which suitable data are available. In this respect, the perspective of rapid progress of ground-based and spatial observations (MOST, CoRoT), is very promising.

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