The beat-curve approach applied to AE UMa

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Abstract

An approach sometimes used to determine the dominant period of a double-mode pulsating star with strong beats in its light curve is to find (O-C) for an observing season by averaging the individual values obtained from observed times of maximum light. The long-term graph of (O-C) versus time can then be used to determine the period and its rate of change. Methods of determining the secondary period usually involve Fourier analysis of relatively complete sets of photometric data.

We show that the beat-curve method, e.g., Elst (1973), Coates et al. (1979, 1982) is an alternative approach also worth considering. The offset of the beat curve usually yields a more precise value of (O-C) for a season than a simple average does, the phase of the beat curve leads to an estimate of the secondary period without the need for intensive photometry, and the amplitude of the beat curve provides information about the relative amplitudes of the modes of oscillation of the star.

Individual Objects: AE UMa, VZ Cnc

Introduction

Because of beating between the fundamental and overtone oscillations for several double-mode pulsating stars, their magnitudes at maximum light and the times of maximum light differ from those they would have if the star only had one mode of oscillation. To a good approximation the discrepancy Δt in the time of maximum light varies sinusoidally with respect to the phase of the beat envelope at which the fundamental maximum falls.

One may construct beat curves in the form

$$\Delta t = B + A \sin 2\pi(\phi_B + \delta) \tag{1}$$

where $\phi_B = EP_0/P_b$, P_b is the beat period and E is the cycle number since epoch. Full details of this beat-curve analysis are given in Coates et al. (1979, 1982), but in summary, changes in the mean level, or estimate of (O-C) for the season (B) and phase of the sinusoidal curve δ, reflect changes in the fundamental and beat periods of the star, from which changes in the overtone period can be derived. In addition, as noted in Landes et al. (2007), the relative phase of the changes in B and in δ may allow us to exclude a light-time effect in a binary as an explanation in cases where (O-C) is changing sinusoidally.

We first compare the precision in (O-C) derived by simple averaging and by the beat-curve approach. Then we apply the beat-curve method to published times of maximum light for the double-mode pulsating star AE UMa, and compare the results with those obtained by Pócs & Szeidl (2001).

Precision in (O-C) found from averaged observations and from beat-curve fitting

In what follows we have assumed that the times of maximum light are determined with perfect precision, because at this point we are interested only with the contribution to the final uncertainties of the number of observations made per season. The actual uncertainties in more realistic cases can easily be calculated if the actual precision of the observations is known.

The RMS value of the sin term in equation 1 is $A/\sqrt{2}$, so that for n observations in a season, the standard error of the mean value of B is $A/\sqrt{2n}$.

We used computer simulations to find the standard deviation in B obtained from nonlinear least-squares fitting to a season's beat curve as a function of n. To a good approximation, we find this to be $4.8 \times 10^{-6} \exp(14/\sqrt{n})$ d, independent of A for A in the range 0 to 0.0075 d (which covers the range of values found for most short-period stars whose light curves exhibit beats).

Figure 1 shows graphs of the standard errors for the two methods above, assuming that A is 0.0030 d in each case. Except for unrealistically small values of n, it appears that there is a significant advantage in estimating B from beat-curve fitting.

Theory for the case of periods undergoing constant rates of change

The following summarises the methods given by Coates et al. (1982) and Landes et al. (2007) to derive periods at epoch and their (constant) rates of change using the beat-curve approach.

We assume that the fundamental and overtone periods are varying at constant rates. Then (O-C) (B in equation 1) and δ will vary quadratically with time:

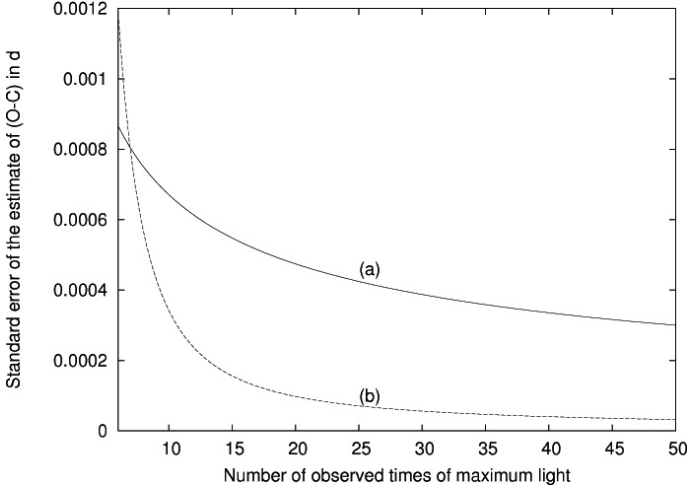$$B = a_B t^2 + b_B t + c_B \tag{2}$$

Figure 1: Standard errors (d) in estimates of B: (a) from straightforward averaging and (b) from beat-curve fitting. The amplitude A of the beat curve is taken to be 0.0030 d in each case.

$$\delta = a_\delta t^2 + b_\delta t + c_\delta \tag{3}$$

where t is time (y) since epoch.

The parameters a_B, b_B, c_B and $a_\delta, b_\delta, c_\delta$ enable us to calculate the best estimates of P_0 and P_1 at epoch, and their rates of change (assumed constant). We let α_0 and α_1 be the true fundamental and overtone periods at epoch, and β_0 and β_1 be their constant rates of change. Then the periods at any time t since epoch will be

$$P_0(t) = \alpha_0 + \beta_0 t \tag{4}$$

$$P_1(t) = \alpha_1 + \beta_1 t \tag{5}$$

The beat period at any time, $P_{\mathsf{b}}(t)$, can be shown, to a good approximation, to be

$$P_{\mathsf{b}}(t) = \alpha_{\mathsf{b}} + \beta_{\mathsf{b}} t \tag{6}$$

where

$$\alpha_{\mathsf{b}} = \frac{\alpha_0 \alpha_1}{(\alpha_0 - \alpha_1)}, \quad \beta_{\mathsf{b}} = \frac{\alpha_0^2 \beta_1 - \alpha_1^2 \beta_0}{(\alpha_0 - \alpha_1)^2} \tag{7}$$

The true periods at epoch, α_0 and α_{b}, and hence α_1, are found from the gradients of the fitted functions to B and δ, evaluated at epoch:

$$\frac{\mathrm{d}B}{\mathrm{d}t} = 2a_B t + b_B = b_B \text{ at epoch} \tag{8}$$

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = 2a_\delta t + b_\delta = b_\delta \text{ at epoch} \tag{9}$$

Now, for the periods $P_0(t)$ and $P_b(t)$ at any time, we have

$$P_0(t) = P_0(\text{ass}) \left[1 + \frac{\text{d}B/\text{d}t}{365.2563} \right] \tag{10}$$

$$P_b(t) = P_b(\text{ass}) \left[1 + \frac{\text{d}B/\text{d}t - P_b(\text{ass})\text{d}\delta/\text{d}t}{365.2563} \right] \tag{11}$$

where $P_0(\text{ass})$ and $P_b(\text{ass})$ are the assumed values at epoch.

Substituting from equations (6) and (7) into (8) and (9), then differentiating with respect to time, we have

$$\frac{\text{d}P_0}{\text{d}t} = \frac{2a_B P_0(\text{ass})}{365.2563} \tag{12}$$

$$\frac{\text{d}P_b}{\text{d}t} = 2P_b(\text{ass}) \left[\frac{a_B - a_\delta P_b(\text{ass})}{365.2563} \right] \tag{13}$$

Equations (10) and (11) yield β_0 and β_b, and having obtained α_0 and α_1 previously, we can calculate β_1 using equations (5). Note that the calculated rates of change of the periods will be in d y^{-1} if the periods are substituted in d, a_B in d y^{-2} and b_δ in y^{-2}.

Beat-curve method applied to data for AE UMa

A study of the stability of the pulsation of AE UMa was published by Pócs & Szeidl (2001). They used Fourier methods, and non-beat-curve (O-C) analysis on previously published photometric data to conclude that the fundamental period had remained essentially constant in the past 60 years. They found that the overtone period changes at a constant rate of $(\frac{1}{P_1})(\frac{\text{d}P_1}{\text{d}t}) = -7.3 \times 10^{-8} y^{-1}$.

We have used the same times of maximum light for AE UMa as those given in Pócs & Szeidl (2001), apart from those based on visual or photographic observations. To these we have added data since published by Agerer & Hübscher (2003), Hübscher et al. (2005) and Klingenberg et al. (2006). We constructed beat curves for each observing season using the same elements as those given by Pócs & Szeidl (2001): $C = 2442062.5824 + 0.08601707E$, $P_b = 0.2936323$ d. Figures 2 and 3 show the values of (O-C) ($= B$) and of δ as a function of time, together with quadratic fits made using singular value decomposition (SVD) with weighted points as described by Press et al. (1992), and also piecewise-linear fits. The error bars in Figures 2 and 3 represent formal statistical errors.

Tables 1 and 2 give results for the quadratic fitting functions and the resulting calculated values of the periods and their rates of change. The most recent published results for the periods and their rates of change are by Pócs & Szeidl (2001), and in Table 3 we compare the rates of change of the periods from our study with those of these authors. We have quoted the most precise values of the periods given by Pócs & Szeidl (2001): in the case of P_0 the value from their (O-C) method, and in the case of P_1 from their fits to Fourier phase diagrams. Their values of the rates of change of the periods come from fits to Fourier phase diagrams. The Fourier analysis was of a large number of photometric data spanning the years 1974 to 1998, see Pócs & Szeidl (2001).
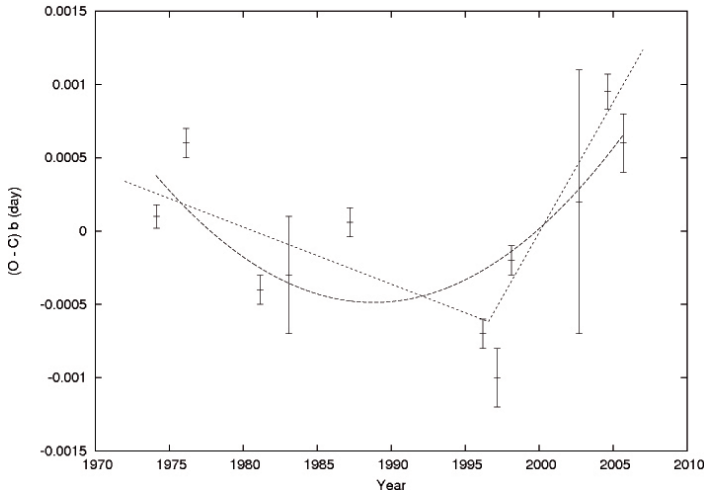
Figure 2: Data for beat-curve mean levels versus time, together with quadratic and piecewise-linear fits.
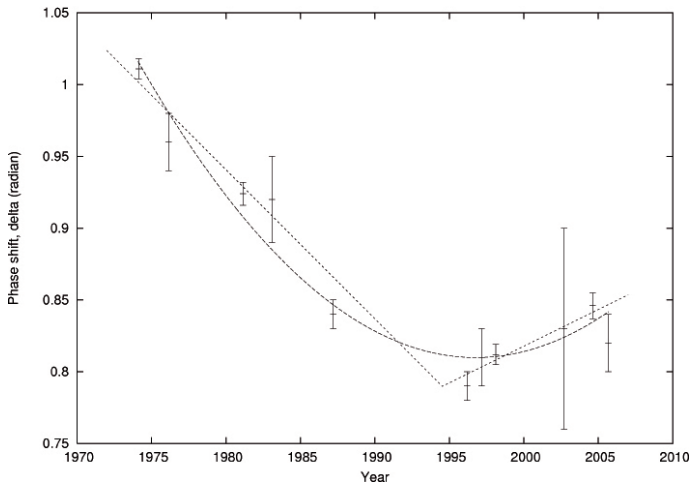

Figure 3: Data for beat-curve phase shifts versus time, together with quadratic and piecewise linear fits.
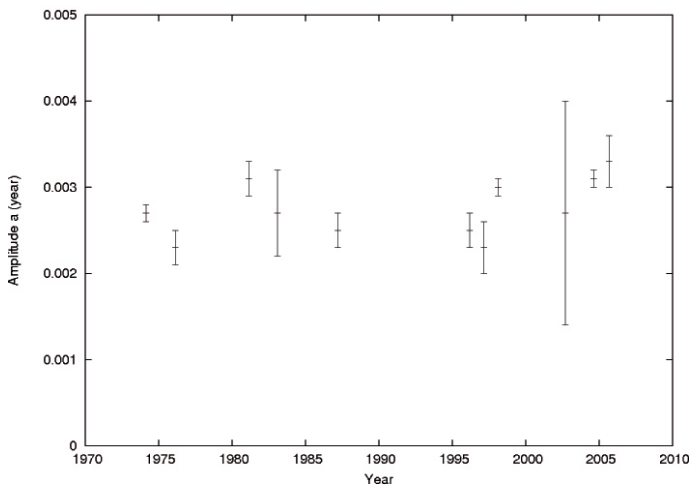
Figure 4: Data for beat-curve amplitudes versus time.

Table 1: Fitting coefficients a_B, b_B, c_B in $B = a_B t^2 + b_B t + c_B$ and $a_\delta, b_\delta, c_\delta$ in $\delta = a_\delta t^2 + b_\delta t + c_\delta$, where t is time (y) since epoch (1974.04).

a_B	$+(4.0 \pm 0.4) \times 10^{-6}$ d y^{-2}
b_B	$-(1.2 \pm 0.1) \times 10^{-4}$ d y^{-1}
c_B	$+(3.9 \pm 0.7) \times 10^{-4}$ d
a_δ	$+(4.0 \pm 0.4) \times 10^{-4}$ y^{-2}
b_δ	$-(0.018 \pm 0.001)$ y^{-1}
c_δ	$+(1.018 \pm 0.006)$

Figure 4 shows the graph of beat-curve amplitudes versus time. In this case there appears to be no clear trend with time, although there is perhaps a tendency for the amplitude to increase from about the mid 1990s. If this were so, it would indicate that the ratio m_1/m_0 increased from this time, where m_1 is the amplitude of the overtone signal and m_0 is the amplitude of the fundamental signal. It is probably too early to draw conclusions using the beat-curve approach about the trend in this amplitude ratio. Pócs & Szeidl (2001) conclude that the amplitudes of the fundamental and overtone oscillations underwent very minor changes over the period 1970 to 1998 for which data were available. Our results for the same period, although scattered, are consistent with this conclusion.

Possible jumps in the periods.

Figures 2 and 3 also show possible piecewise linear fits to the data. These fits were found by minimising χ^2 for the pairs of lines for B and δ. Using the notation of Coates et al. (1979), we let the equations of the best-fitted lines be:

pre- 1996

$$B = b_1(year) + a_1 \tag{14}$$

$$\delta = b_1^{'}(year) + a_1^{'} \tag{15}$$

post- 1996

$$B = b_2(year) + a_2 \tag{16}$$

$$\delta = b_2^{'}(year) + a_2^{'} \tag{17}$$

The true values of P_0 are calculated in the usual way using the parameters b_1 and b_2. Calculating the true values of P_b is slightly complicated by the fact that departures of *both* P_0 and P_b from their assumed values would contribute to a change in δ. A fairly straightforward analysis leads to the expression:

$$P_b(true) = P_b(ass)\left\{1 + \frac{b - b^{'}P_b(ass)}{365.2563}\right\} \pm \left\{\left(\frac{P_b(ass)}{365.2563}\right)^2\left[\sigma_b{}^2 + (P_b\sigma_b{}^{'})^2\right]\right\}^{\frac{1}{2}} \tag{18}$$

Table 3 gives the gradients of the fitted straight lines together with the resulting periods pre and post-1996. The fractional changes in P_0 and P_1 in 1996 were $\Delta P_0/P_0 = +6 \times 10^{-7}$ and $\Delta P_1/P_1 = -2 \times 10^{-6}$.

Conclusions

The beat-curve approach has enabled us to deduce detailed and quite precise information about the fundamental and first overtone periods of AE UMa without the necessity for Fourier analysis of long spans of photometric data. We confirm the values found by Pócs & Szeidl (2001) for the rates of change (assumed constant) of the periods, which are similar in magnitude to those of other Pop. I radially pulsating delta Scuti stars (Breger & Pamyatnykh 1998). In addition, because we have access to times of maxima for the star post-1998, we have been able to extend the work of Pócs & Szeidl (2001) and deduce that there were possible sudden jumps in both the periods in approximately 1996, thus adding AE UMa to VZ Cnc (Arellano Ferro et al. 1994) as radially pulsating delta Scuti stars which have possible sudden jumps in period.

Although the beat-curve method can reveal no information about the low-amplitude modes in pulsating stars, for which accurate photometry and Fourier methods are needed, it can make good use of the many accurate times of maximum light, increasingly being published, to investigate and determine the fundamental and overtone frequencies of double-mode pulsating stars.

Table 2: Periods (in d) at epoch (1974.04), their rates of change $\mathrm{d}P/\mathrm{d}t$ (in d y^{-1}) and $(1/P)\mathrm{d}P/\mathrm{d}t$ (in y^{-1}), assuming that the rates of change are constant.

	Present study	Pócs and Szeidl (2001)
P_0	0.0860170421(7)	0.086017053(6)
P_1	0.06652840(1)	0.06652836(4)
P_b	0.2936365(3)	0.2936356(7)
$\mathrm{d}P_0/\mathrm{d}t$	$+(1.9 \pm 0.2) \times 10^{-9}$	$+(1 \pm 1) \times 10^{-10}$
$\mathrm{d}P_1/\mathrm{d}t$	$-(8.3 \pm 0.9) \times 10^{-9}$	$-(5 \pm 3) \times 10^{-9}$
$\mathrm{d}P_\mathrm{b}/\mathrm{d}t$	$-(1.8 \pm 0.2) \times 10^{-7}$	$-(1 \pm 1) \times 10^{-7}$
$(1/P_0)\mathrm{d}P_0/\mathrm{d}t$	$+(2.2 \pm 0.2) \times 10^{-8}$	$+(1 \pm 1) \times 10^{-9}$
$(1/P_1)\mathrm{d}P_1/\mathrm{d}t$	$-(1.2 \pm 0.1) \times 10^{-7}$	$-(8 \pm 5) \times 10^{-8}$

Table 3: Gradients of the fitted straight lines and the calculated constant periods pre- and post-1996.

	Pre-1996	Post-1996
b	$-(3.9 \pm 0.5) \times 10^{-5}$ d y^{-1}	$+(1.8 \pm 0.2) \times 10^{-4}$ d y^{-1}
b'	$-(1.04 \pm 0.05) \times 10^{-2}$ y^{-1}	$+(5 \pm 1) \times 10^{-3}$ y^{-1}
P_0	0.086017061(1) d	0.086017112(4) d
P_1	0.066528321(6) d	0.06652817(2) d
P_b	0.2936347(1) d	0.2936312(3) d

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