ON THE USE OF THE FOURIER NUMBER TO INTERPRET THERMAL MEASUREMENTS WITH A QUASI–LINEAR HEAT SOURCE

W. Marczewski*, B. Usowicz†, B. Dabrowski*, R. Wawrzaszek*, K. Seweryn*, E. Sendek‡, N.I. Kömle§, G. Kargl§

Abstract

This paper describes a way of calibrating a thermal probe for the *Quasi–Linear–Heat–Source* (QLHS) method. The heat source can be a short and thick torus, not necessarily a long and thin wire. The source is an analog of the MUPUS–TP probe for determining the thermal conductivity. Similar probes were also employed for investigating terrestrial media on Earth. Two reference tests are required for calibration. The first test is for normalizing apparent properties to the known media properties of PTFE (Teflon). The second test is for scaling the magnitude of the properties, in order to cover the range between PTFE and water ice. Then measurements are interpretable in absolute property values, even when a geometry significantly deviating from that of a thin wire is used. In general the temperature response of a sensor to heating is similar to the function obtained with a thin wire. The difference is mainly the value of apparent slope, observed in the log–time domain, and the way of interpreting it. The conclusion is that the processes measured must be referred to some known properties of two kind of media. The comparison is founded on similarity of processes, expressed by the Fourier number (Fo). This calibration method enables assessing the specific heat capacity from a routine test. A reasonably good agreement of derived thermal properties has been demonstrated by measurements in snow and soil.

1 Introduction

The *Linear Heat Source* (LHS) method is widely known as the so–called *Hot Wire* method, and is a standard, defined by ASTM (D 5334-00, and IEEE Std 442–1981). It is related

* *Space Research Centre, Polish Academy of Sciences, ul. Bartycka 18A, 00-716 Warsaw, Poland*

† *Institute of Agrophysics, Polish Academy of Sciences, ul. Doswiadczalna 4, P.O. Box 201, 20-290 Lublin 27, Poland*

‡ *Technical University, Applied Mathematics, Al. Tysiaclecia Panstwa Polskiego 7, 25-314 Kielce, Poland*

§ *Space Research Institute, Austrian Academy of Sciences, Schmiedlstrasse 6, 8042 Graz, Austria*

to a long and thin cylinder configuration as a heat source. The LHS method is well established in theory since the work of Carslaw and Jaeger (1959).

This paper deals with the tube shaped probe used for the cometary experiment MUPUS (Spohn et al., 2006, Seiferlin et al., 1996; Kömle et al., 2002; Marczewski et al., 2004; Hagermann et al., 1999). The probe is a series of 16 short torus–shaped elements, which can be simultaneously sensed and heated. Matching that configuration to the shape of a quasi–infinite cylinder is not possible in this case. The probe was aimed for measuring thermo–physical properties in non–homogeneous media (Seiferlin et al., 1996).

The principle of the heat impact, applied for determining thermal properties of media, is universal. Early attempts to validate the proposed method proved relevance and feasibility of the probe (Banaszkiewicz et al., 1997). Then, further works disclosed that a real challenge, not only for modelling, is to find a reliable method of interpreting results. Despite relevance and a great potential of those works, the capability of estimating thermal properties from tests was still limited and not satisfactory. Matching a transient response from the model to the response from a test involves so many boundary conditions and parameters, which must be incorporated into a finite element model (FEM), that their control in tests becomes problematic. Currently the concern is to interpret properties exclusively from the test data. It does not aspire yet to assess final precision, but to introduce a new point of view.

It was a particular need of MUPUS to define a method capable to assist this experiment. The QLHS (*Quasi Linear Heat Source Method*) may serve this purpose. The tests in this work are focussed on media commonly available on Earth. The test media were water ice, snow, soil and PTFE. The interest in soil media was also motivated by available confrontation of measured properties to the indirect method of defining thermal properties, developed by Usowicz et al. (2006) for agro–physical purposes. This work is an extension of two programs. One is the cometary experiment MUPUS. The other is the spill–over program EXTASE for Earth science purposes (Schröer, 2006). Both programs are conducted by T. Spohn, WWU, Muenster. The cometary experiment is to investigate the ground of the comet in the mission *Rosetta*, after landing of the *Philae* lander in 2014. Recently the work has been extended by another program — *Soil, Water and Energy Exchange* (SWEX), being a part of the ground calibration–validation campaign Cal–Val, held by ESA SVRT (SMOS Validation and Retrieval Team) for the ESA Water Mission SMOS (*Soil Moisture and Ocean Salinity*).

2 The probe and the method

The probe is a tube with 16 sensing and heating elements, as illustrated in Figure 1. The elements are bonded to the 1 mm thick tube wall from inside. The thermal conductivity of the tube material (glass fiber) is roughly known as 0.5 W/m/K. The whole probe is approximately 32 cm long. The shortest element is 9 mm and the longest is 40 mm long. The outside diameter of the tube is 10 mm. For terrestrial applications the probe can be placed into soft soils by pushing it manually into the ground, or in the case of harder material by pushing it into a pre–drilled hole. For the cometary experiment the probe is inserted into the ground by a complex hammering device. The probe elements provide sensing and heating. In the sensing mode the material temperature profile and its variation over the length of the tube is measured. In the active mode individual elements are heated with a defined power over a defined time span. The temperature response of the sensor contains information on the thermal properties of the surrounding medium in a particular depth. Their proper derivation is the main topic of this paper.

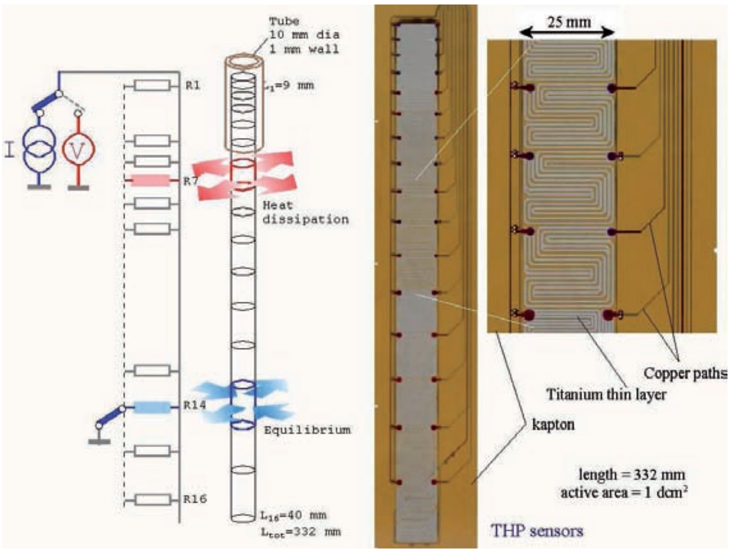

Figure 1: General scheme of the MUPUS/EXTASE thermal probe and the view of sensors, when they are not yet integrated inside the tube. The tube is depicted schematically by the cylindrical section at the top of the probe.

The entire side area of the probe (≈ 1 dm^2) is covered by thermally active elements. The larger the area of sensors, the better temperature gradient values can be determined. The larger the area of heaters, the better is heat dissipation. Electrically, the elements are temperature dependent resistors (RTD), sampled when sensed or supplied with electric power when heated. The method is derived from the LHS (*Line Heat Source*) or the HW (*Hot Wire*) method.

We started interpreting properties in the way provided by Healy et al. (1976), disregarding the assumptions on large length and small thickness, which are not fulfilled for the QLHS method. The temperature rise function upon heating is given in Carslaw and Jaeger (1959) as

$$\Delta T = \frac{q}{4\pi\lambda} \ln\left(\frac{4\kappa t}{a^2 C}\right) \qquad (1)$$

where q is the heat source term, in terms of power density per unit length [W/m], κ is the diffusivity in [m^2/sec], λ is the thermal conductivity of media, a is the radius of the wire, t is time in linear measure [s] after start of heating, and C is the calculated from Euler's constant γ as $C = \exp(\gamma) = 1.7811$.

This equation predicts $\Delta\mathrm{T}(t)$ as a function of the diffusivity κ and the conductivity λ. It describes the transient response on a steady heat stimulus switched abruptly on. $\Delta\mathrm{T}$ approaches a linear slope when plotted as a function of $\ln(t)$. Negative values of ΔT for small arguments under the logarithm do not reach a physical sense. The argument under the natural logarithm in equation (1) can be expressed in terms of the Fourier number (Fo), which is a dimensionless number defined by the thermal properties and time as

$$Fo = \frac{\kappa t}{a^2} = \frac{\lambda t}{\rho_b c_h a^2} \tag{2}$$

where ρ_b is the specific bulk density of themedium, c_h is the specific heat capacity of medium, and a^2 is the geometric dimension related to the heat source.

For a truly thin wire, the Fourier number falls in the range of thousands, due to the small radius a of the wire. If the diffusivity κ does not change much with temperature T, under a moderate heating, Fo follows the elapsed time uniformly. Sometimes, Fo is simply called the dimensionless time, or a universal similarity measure to compare processes in different media. It serves comparing the processes, not media, quantitatively. The means for comparing, are modest, however. Two thermal processes are equivalent when their corresponding Fourier numbers are equal.

Healy employed Eq. (1) to determine the thermal conductivity λ as

$$\lambda = \frac{q}{4\pi} \frac{d\ \ln(t)}{d(\Delta T)} \tag{3}$$

where λ is the thermal conductivity in [W/m/K], q is the heat generation, in terms of power density per unit length [W/m].

The precision of practical determining the conductivity from equation (1) depends on the argument value $4\kappa t/(a^2 C) \geq 1$. The same way obeys the equation(3), though the argument variable $4\kappa\mathrm{t}/\mathrm{a}^2\mathrm{C}$ is not distinguished explicitly. Both, the conductivity and the diffusivity, are determined from the same response function.

As shown by Nagasaka et al. (1981), the diffusivity κ can be determined from the same test data as:

$$\kappa = \frac{a^2 C}{4t}\ \exp\left[\frac{d\ \ln(t)}{d(\Delta T)} \Delta T\right] \tag{4}$$

Nagasaka provided also a simplified method of finding the diffusivity from a line heat source measurement. The slope of the temperature rise may be expressed as

$$\Delta T = A\ \ln(t) + B \tag{5}$$

where A is the slope, and B is the temperature rise at $\ln t = 0$ (i.e. 1 s after the begin of heating, if time is measured in seconds, since $\ln 1 = 0$). The expression for the diffusivity equation (4) can then be written in terms of the constants A and B as

$$\kappa = 0.4453\, a^2 \ \exp\left(\frac{B}{A}\right) \tag{6}$$

With the above equations the diffusivity and the conductivity can be derived independently from the same data set. Although Healy's formulae are strictly valid only for the LHS case, we try to apply them also on measurements with the MUPUS/EXTASE probe and check *a posteriori* inhowfar this gives consistent results.

3 Tests

The media used in our tests were the following:

1. PTFE — tested in ambient temperature +20°C.
2. ICE — tested at -45°C.
3. SNOW2 — with a water content of about 36.7%, in volumetric measure.
4. SNOW1 — with a water content of 19.1%.
5. SOIL — in natural open field conditions, but the thermal conductivity of this site was known to be about 1.9 W $m^{-1}K^{-1}$ at a level of about 10–15 cm below the surface. The thermal properties of the soil were controlled by means of the Usowicz model (Usowicz et al., 2006) using with input data from physical, as well as chemical and granulometric analysis.

The test data set is given in the Appendix to this paper. All sensors were sensed for the temperature, but only a few elements were heated. In the following, only the data for sensor 7 are considered. The heating time was always 900 s and temperature data were recorded every 10 s.

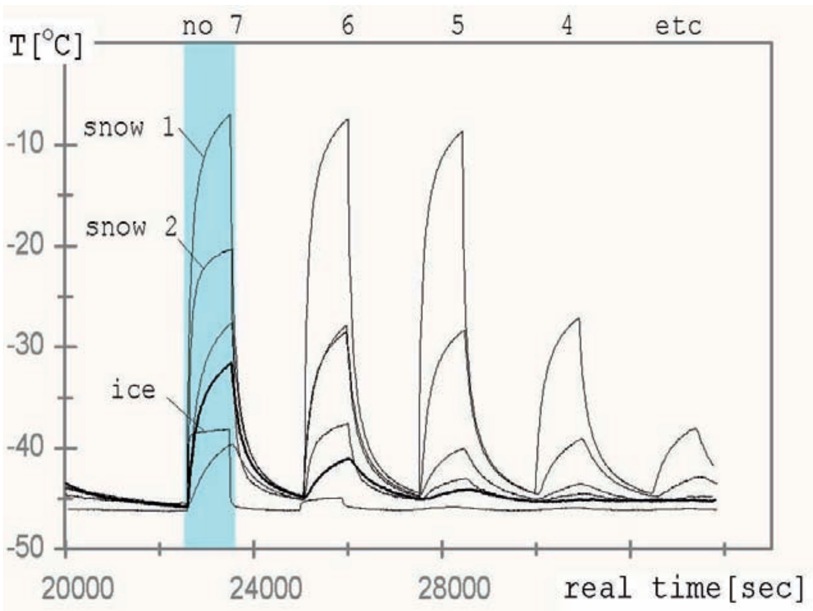

Figure 2: Plots of the temperature responses showing heating intervals on sensor 7, counting from the top of the probe (thick lines), combined for three experiments: (a) "snow1" for snow media with $w_{vc} = 0.191$ *(volumetric water content); (b) "snow2" for snow media with* $w_{vc} = 0.367$*; (c) "ice1" for solid water ice media (hexagonal ice). The media were frozen to* $-45°C$ *to avoid melting modification under heating. Time intervals of heating were 900 s (15 min) long. Sampling intervals were 10 s. Only one sensor in the probe was heated, namely sensor 7 (thick lines). The plots are supplemented by a few plots for some neighbouring sensors. When only one sensor is heated the remaining sensors respond due to the heat received from this sensor. Each experiment was performed separately with the same setup order and plots are combined in a common time scale.*

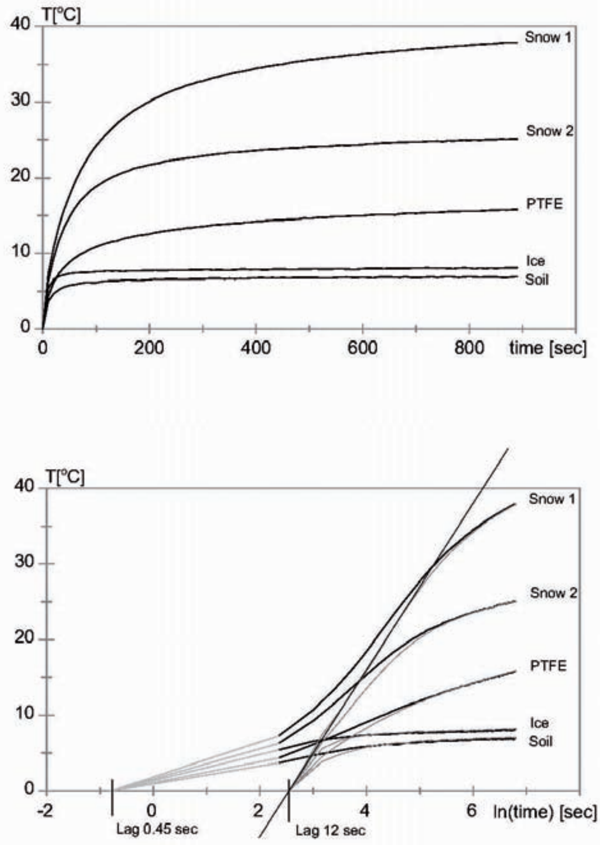

Figure 3: The temperature responses of sensor 7, in different media. The heating time was 900 s with 10 s sampling interval. The upper panel is for the real time domain. The lower panel is for the ln(time) domain. The second plot is done twice — once with a delay lag 0.45 s added to all samples, and again with a lag 12 s.

Figure 3 shows the response of sensor 7 in different media, both as a function of t and as a function of $\ln t$. From the plot one gets the impression that for a reasonably long heating time one always gets a linear slope in the $\ln(t)$ domain, irrespective of the deviating geometry. Of course the initial phase of the temperature increase is nonlinear and it is affected both by the properties of the sensor and by the contact between sensor and medium.

4 General outline of the concept

The first step is to characterize the media by properties according to the case of a thin wire, and to normalize the properties to the known properties of the PTFE reference medium. The second step is converting these apparent values to absolute physical measures, on the basis of a calibration test with two different reference media, PTFE and water ice, which are sufficiently different in their thermal properties to cover the expected range. Thus there are three tests involved, one is the work test with the unknown medium, and two with reference media, whose properties are known. In all phases, the same sensor element is employed, with its entire imperfectness, and only the test data are interpreted.

First one evaluates the conductivity from Eq. (3) and the diffusivity from Eq. (4), as if the thermal probe would be simply a wire. Since it is different, results occur also very much differently. The purpose is to characterize that by referring the apparent properties to the respective properties of the reference media.

5 Calibration

The purpose of calibration is to determine the scale factors A_{scale} and B_{scale}. The calibration tests are performed in the same way as the work tests. The only difference is that media with known properties are used.

The probe measures temperature T and power density q as a function of time.The principle of sensing is based on resistance temperature dependent of sensors (RTD). PTFE was chosen as the first reference medium. Its thermal properties are given in Table 1. Water ice was taken as the second calibration medium, with the property values given also in Table 1.

Table 1: PTFE and ICE thermal properities

Properties Media/Measure	ρ_b [kg/m^3]	c$_h$ [J/kg/K]	λ [W/m/K]	κ [cm^2/sec]	P [J/m^2/K/$\sqrt{sec}$]	c$_v$ [J/cm^3/K]
PTFE@+35^{o}C	2200.00	1136.40	0.27307	0.00109	0.08263	2.50007
ICE@-40^{o}C	920.80	1820.00	2.43035	0.01450	0.20181	1.67586
prop^{ICE}/prop^{PTFE}	0.419	1.602	8.900	13.278	2.443	0.670

Examples of using calibration data are shown in Table 2, with the scale factors A_{scale} and B_{scale} for several tests.

Table 2: Example scaling factors

t_n=180 sec	ICE	SNOW2	SNOW1	PTFE	SOIL
A_{scale}	4.1517	1.1916	1.0516	2.2217	3.5921
B_{scale}	0.9155	0.9155	0.9155	0.9155	0.9155

Values of the apparent normalized properties are shown in Figure 4. They mean, for example, that the PTFE conductivity value is 2.2217 times larger than the specific property. Similarly, the diffusivity appears on the level of 0.9155 of the specific diffusivity. Comparing media directly by these scales, one could expect that the water ice conductivity is only about 2 times larger than the conductivity of PTFE, what is untrue. The conclusion is that comparing media, directly — property by property, is wrong. The scales relate properties of the test material to PTFE properties by apparent, but not by absolute measures. The B_{scale} values occur independent of the media under the tests, what seems a bit peculiar but proves independence of the diffusivity from the conductivity. The B_{scale} values are equal only for the reference time $t_n = t_{refB}$. Figure 4 shows that all other diffusivity values, beyond the reference time, are sensitive to the medium properties.

6 Local versus large scale slope definition

The property definition (3) involves the slope in terms of a derivative. The derivative is naturally associated to the local time scale. The slope, determined from real test data, containing noise components, is usually spread in magnitude. The spread is the larger when the samples are closer. Evaluating differences on distant samples instead, determines the slope in a way less sensitive to noise components. Below the suggested procedure for slope determination is given. It employs a bound of transects, instead of tangents, for investigating the slope.

To interpret the slope, a proper time window must be chosen in the interval. We start with choosing two reference samples. The first reference is the $refA$ sample, for determining the slope A. The choice is not critical, so one can take the first sample after switching the heating power on. Then a series of transects, from the $refA$ sample to all other samples, is generated in a simple loop. The second reference is the $refB$ sample, for determining B, which corresponds to ΔT at $t = 1$ s, see Eq. (5). B serves determining the diffusivity, and is obtained also by a bound of transects, linking the $refB$ sample to any other sample in the interval. The reference samples should be reasonably distant, but the choice is not critical. The choice is even out of importance when the slope is perfectly linear. To obtain thermal properties for all samples, one needs to choose $refB$ for each subsequent sample in the interval. The procedure is simple and may be limited to two samples $refA$ and $refB$, with $refB$ walking through the entire interval. The task may be implemented by a simple loop.

The procedure is run over all samples. The slope A related function f_A has no value at the first sample in the time window, because there is no temperature difference when a current and a reference sample $refA$ are identical. The function f_B, at the end window, has a

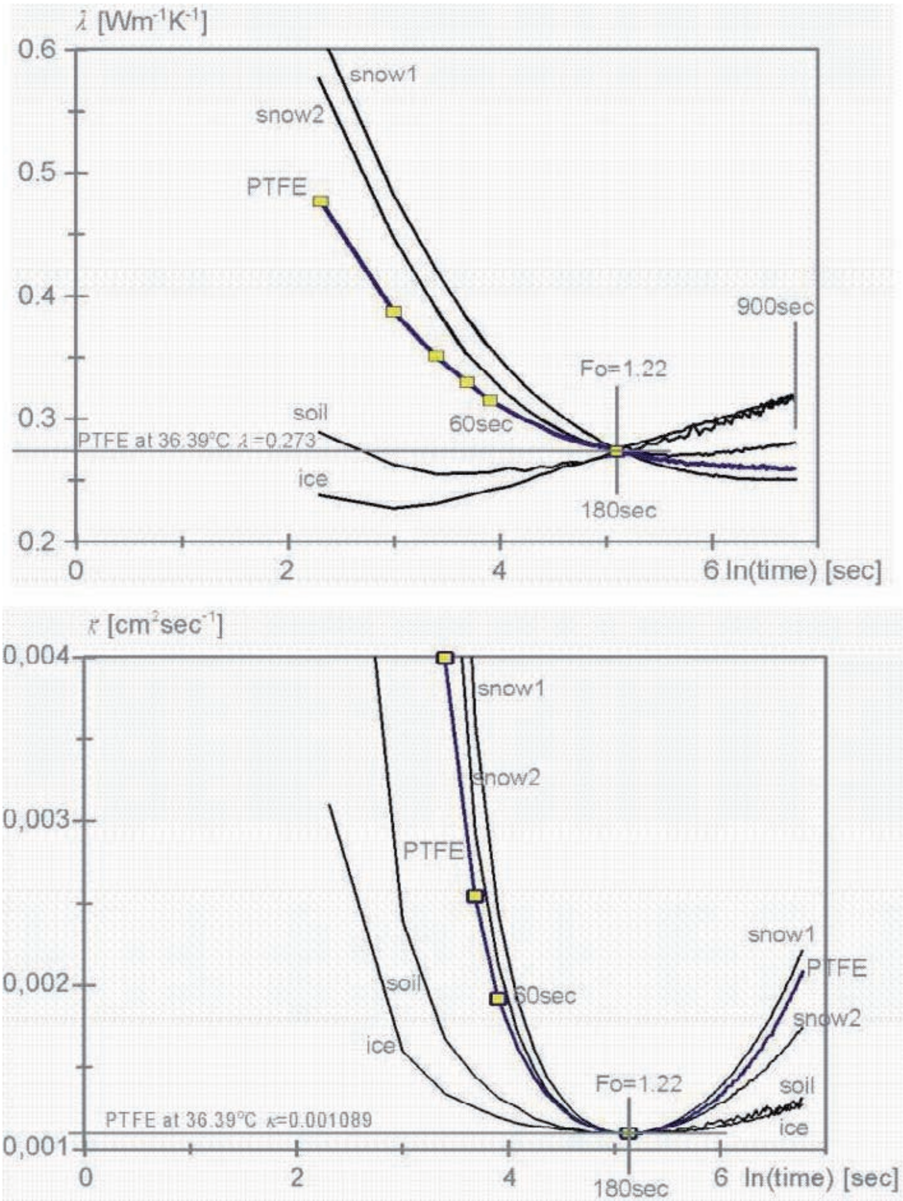

Figure 4: Transient responses from several tests in different media, all normalized to the reference media properties for the sample corresponding to time=180 s.

particular constant value which is independent of the medium (!). That independence of media is peculiar, but is justified. A corresponding difference between the current temperature, at an arbitrary time t_n, and the reference temperature, at the time $t_n = t_{refB}$ is equal zero, what yields $exp(B/A) = 1$, in Eq. (4). The value of $f_B(t_n = t_{refB})$ is non–zero and depends only on the choice of the time $t_n = t_{refB}$. All other values of $f_B(t_n)$ depend on the medium. The scale factors A_{scale} and B_{scale} are determined for the the time $t_n = t_{refB}$, however. Taking the scales is equalizing the Fourier numbers. The scale B_{scale} makes them equal. The scale factors, realize substituting the temperature response by new products of f_A and f_B. They are not final properties, but cover the contents of the expressions Eq. (3) and Eq. (4) and express respective property measures. The scales are determined locally, but the slope related functions, f_A, and f_B, are determined over the entire time window.

7 The slope related function f_A

The slope–related function $f_A(t_n)$ can be determined for all t_n. It is given as

$$f_A(t_n, \frac{1}{A}) = \frac{ln(t_n) - ln(t_{refA})}{T(t_n) - T(t_{refA})} \tag{7}$$

The function $f_A(t_n)$ is calculated on the base of a current sample, at time t_n, and the reference time t_{refA} is now assumed to be the first sample in the data series.

Taking the formula on the conductivity λ from Eq. (3), one gets the apparent value of the property $\lambda_{app}(t_n)$, respecting the heating power density q, expressed by the power value in the test, to the power value in the test with PTFE. Power in absolute measure is not needed at all. Then, the apparent conductivity is required to be scaled by the level specific for PTFE, by means of A_{scale}. Healy scaled the slope to the absolute measure of conductivity by the factor $q/(4\pi)$. This links the result to the heating power density q, and respective constants in Eq. (1). That was the formula Eq. (3), proved for a perfect wire. We take the same relation as Healy, but account that the same constants are covered by the sought scaling factor A_{scale}. Only the power density is introduced directly from the test. The scale factor must be determined based on PTFE. That is done at the selected time t_n.

$$\lambda_{app}(t_n) = \frac{q_{TEST}(t_n)}{q_{PTFE}(t_n)} f_A(t_n) = \lambda_{PTFE} A_{scale} \tag{8}$$

From that we know the scale A_{scale}, and the apparent conductivity values for all t_n. One can do that for all $t_n = t_{refB}$, and determine scale factors for the entire interval.

8 The position related function f_B

Similarly, we proceed with the function $f_B(t_n)$, determining the position B of the slope A and taking samples of data for all t_n.

$$f_B(t_n, \exp(\frac{B}{A})) = \exp\left[(\ln t_{refB} - \ln t_n \frac{T(t_{refB}) - T(t_n)}{\ln(t_{refB}) - \ln(t_0)}\right] = 1 \quad |t_n = t_{refB} \qquad (9)$$

This expression is also the same as for the wire. It involves the position B and the slope A, in the way following Nagasaka. For determining the position B, the time t_0 is assumed to be 1 sec, to get the value of $\ln(t_0)$=0. The position B is a current estimation of the temperature increase, based on the abscissa coordinate for t_0, not on the temperature before heating. Each time, the values B and A are computed for the current sample time t_n, and the reference time of the time window t_{refB}, for all t_n. The functions $f_A(t_n)$ and $f_B(t_n)$ do not represent the properties. They are an apparent record of the test history, or two aspects of the response shape.

The same is done with the new data series $f_B(t_n)$, which is scaled, or normalized by B_{scale}, to obtain the apparent $\kappa_{app}(t_n)$

$$\kappa_{app}(t_n) = f_B(t_n) = \kappa_{PTFE} B_{scale} \qquad (10)$$

The procedure is summarized in Figure 5.

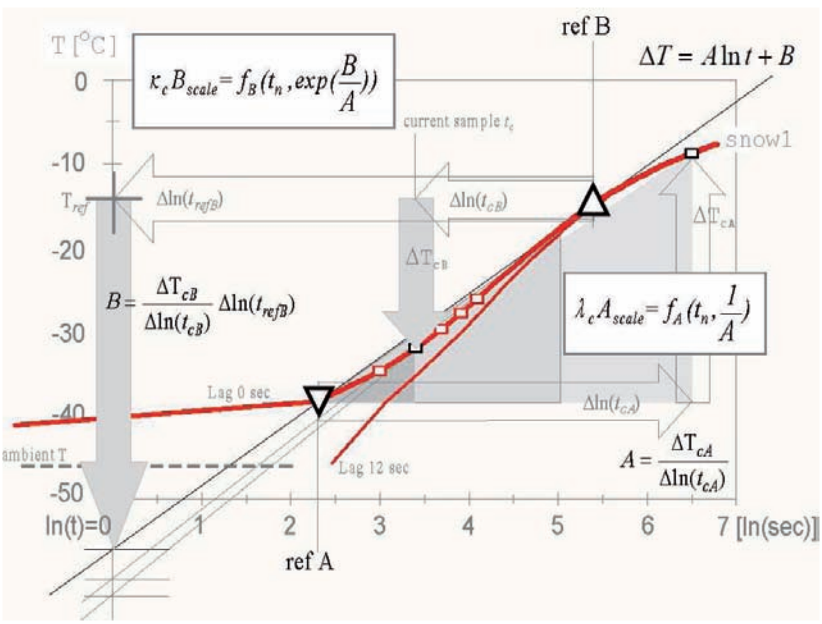

Figure 5: Illustration of the slope A and its position B, determined from the same test data, independently.

The example scales can be very much different from unity and only the scaling of the conductivity varies with media. The scale for the diffusivity does not depend on power density, and does not depend on the medium. That is peculiar because different media have their own specific diffusivity values. However an absolute measure of the specific diffusivity is determined by the scaling factors κ_{scale} determined by the test data, and by the factor $0.4453a^2$, covering dependence on geometry.

9 Conversion of apparent properties to absolute measures

All properties in apparent forms for all tests, are displayed in Table 3. All media are assigned to the specific heat of PTFE.

Table 3: Apparent thermal properties, in all the test media, obtained by of normalizing thermal properties to PTFE specific values

Properties Media	P_{app} [J/m^2/K/$\sqrt{sec}$]	λ_{app} [W/m/K]	κ_{app} [cm^2/sec]	Fo_{app} [−]	$c_{v_{app}}$ [J/cm^3/K]
ICE	0.154404	0.953592	0.003814	3.492065	2.50007
SNOW2	0.044325	0.078586	0.000314	0.287783	2.50007
SNOW1	0.039111	0.061184	0.000245	0.224056	2.50007
PTFE	0.082626	0.273074	0.001092	1	2.50007
SOIL	0.133590	0.713826	0.002855	2.614042	2.50007

The apparent value of the thermal inertia $P_{appTEST}$ for the work test media, was based on the absolute inertia of PTFE, taken as in the Table 1.

$$P_{appTEST} = \frac{A_{scaleTEST}}{A_{scalePTFE}} P_{PTFE} \tag{11}$$

The apparent value of the diffusivity $\kappa_{appTEST}$ for the work test media was based on the absolute inertia of PTFE, and on the specific heat of PTFE c_v, taken as in the Table 1.

$$\kappa_{appTEST} = \left(\frac{P_{appTEST}}{c_{vPTFE}} \right)^2 \tag{12}$$

The apparent value of the specific heat per unit volume $c_{vappTEST}$ for the work test media was evaluated as

$$c_{appvTEST} = \frac{\lambda_{appTEST}}{\kappa_{appTEST}} \tag{13}$$

The apparent values of all properties, are now effects of normalisation to PTFE properties, and therefore, the values c_{vTEST} for all media, are equal exactly to c_{vPTFE}.

The absolute value of the Fourier number Fo_{TEST} was evaluated as determined by the ratio of $\kappa_{appTEST}$ to the absolute value κ_{PTFE},

$$Fo_{TEST} = Fo_{appPTFE} \frac{\kappa_{appTEST}}{\kappa_{appPTFE}} \tag{14}$$

Figures 6 and 7 show the apparent values of conductivity and diffusivity as a function of the Fourier number and of the logarithm of time, respectively. Table 4 displays all properties.

Table 4: Thermal properties for all the tests, in absolute measures

Properties Media	P_{abs} [J/m^2/K/$\sqrt{sec}$]	λ_{abs} [W/m/K]	κ_{abs} [cm^2/sec]	Fo_{abs} [−]	$c_{v_{abs}}$ [J/cm^3/K]
ICE	0.201815	2.430347	0.014502	13.277119	1.675856
SNOW2	0.019026	0.133519	0.004925	4.508725	0.271121
SNOW1	0.010368	0.075247	0.005268	4.822673	0.142848
PTFE	0.082626	0.273074	0.001092	1	2.50007
SOIL	0.167252	1.653808	0.009777	8.95155	1.69145

10 Results and discussion

Now, the property values are believed to be free of effects caused by unintended inconsistency between properties. The tests performed in ice and PTFE (Table 4), are represented exactly by the values from Table 2, because they were taken as references. The other media like soil and two sorts of snow, diversified by water content (19% for SNOW1, and 37% for SNOW2), are sorted well by properties. Also the Fourier numbers indicate that the water ice media exhibit the largest diffusivity.

The last column in Table 4 (volumetric heat capacity) also discloses a realistic behavior in all cases. Specific heat for the water ice and PTFE are exact, as expected from reference data. The specific heat of soil is also realistic, because it is known from independent measurements. The volumetric soil moisture was about 25% at that site. It means that $1/4^{th}$ of a cubic centimeter of soil contained water. If the entire content were water, then the specific heat would beequal to 4.18 [J/cm^3/K]. The value of 1.69 [J/cm^3/K], fits well the soil moisture.

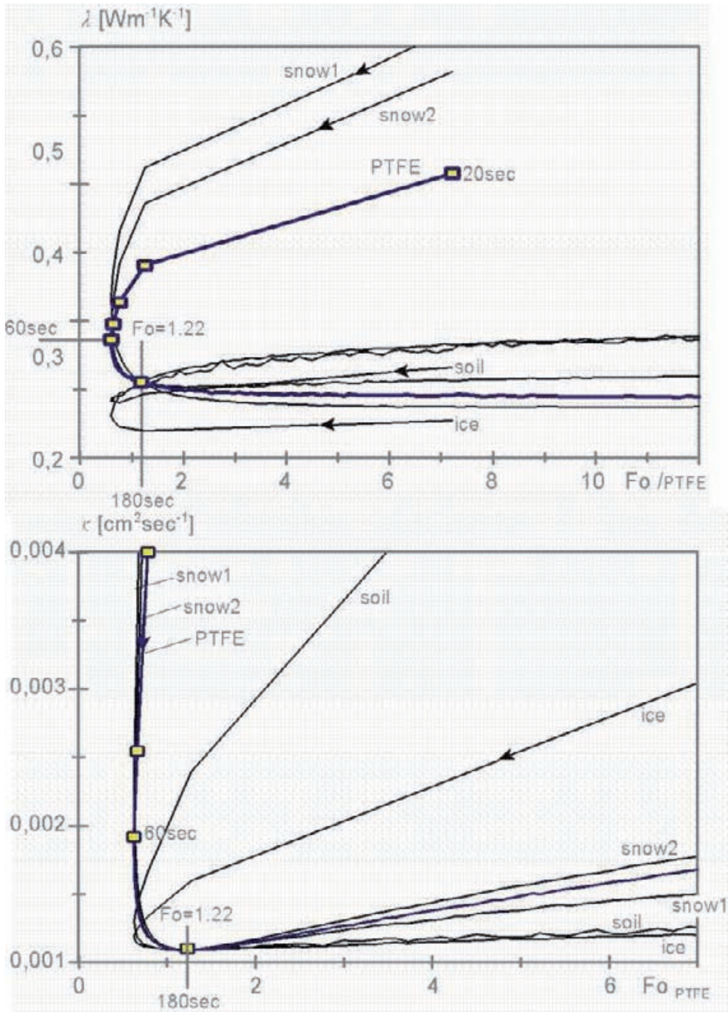

Figure 6: Apparent thermal conductivity λ and diffusivity κ, for different media, in normalized values versus Fo. All media were equalized to the specific properties λ and κ of PTFE, thus they all have a common domain of Fo, proper to PTFE. The first few PTFE samples (10 s each) are depicted by blocks, to show how they are close to the target specific value after 60 s since the power was set on. The first sample in the test was taken for the reference $refA$. The reference $refB$ was taken for the time 180 s after the start in this plot. The temperature of PTFE in this test was $36.4°C$. Snow and ice media were tested frozen to $-45°C$. Each test was performed independently, in different temperature conditions.

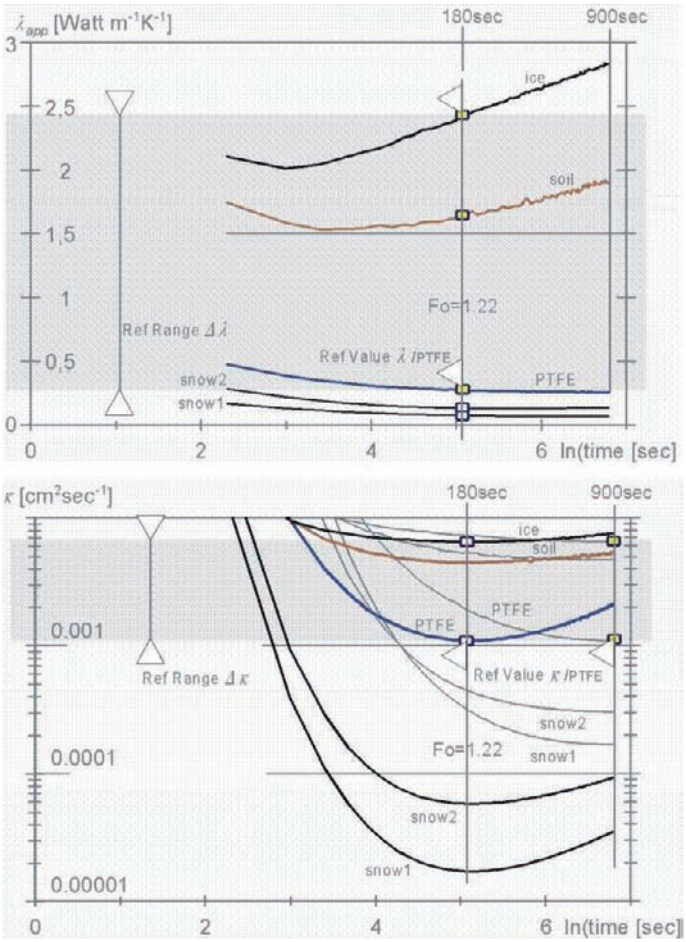

Figure 7: Apparent thermal conductivity (top) and thermal diffusivity κ (bottom), for different media, in de-normalized values versus log-time. The square blocks show where the normalisation was done – in the time 180 s after the beginning of the interval. Only that values represented by rectangle blocks are formally exact. Grey bands in the background, distinguish the range of a respective property, taken for a reference for λ (on the left panel), and for κ (on the right panel), supported by specific properties of PTFE and water ice. The square block for PTFE, is pointed by a triangle, to distinguish which property was a base for evaluating all other properties. Two triangles on the diffusivity plot (panel right) show that the reference value of the diffusivity was shifted with the change of the $refB$ from 180 s to 900 s, though it is the same property value of PTFE.

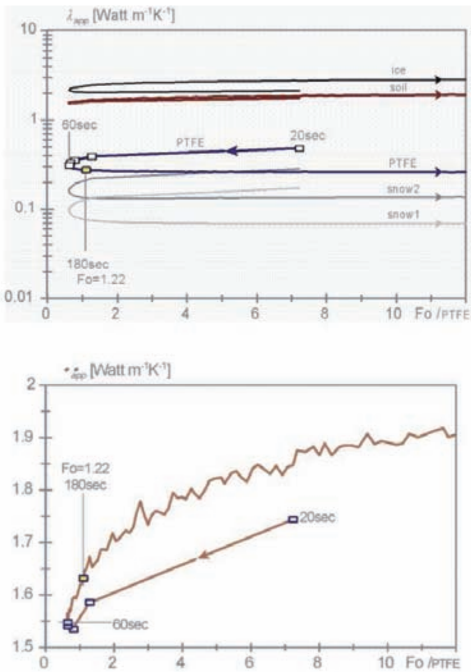

Figure 8: Apparent thermal conductivity in re–normalized values, versus Fo. The values are plotted twice. The upper panel is in the log–scale of the conductivity, to show how clear is the segregation of media by the property. The right panel is in a linear respective scale, to show that good determination of an apparent property, and a random signature of noise from the temperature response. The dense media provided the temperature rise responses with a very narrow temperature range, what didn't disturb determination of properties. Data for the soil media, was mostly contaminated by noise components, due to instability of media in open field conditions. The measured values are taken for the time 180 s, determined with the Fo=1.22. Low Fo proves that in the time, the process is in an early phase under development. The time even 2 or 3 times earlier, should also provide a reasonable property, if only one decides to normalize the response then. The arrows in characteristics, indicate the direction of the time flow from start to end.

The soil media agree with the values predicted by the model of Usowicz moderately. The model predicts 1.9 W/m/K, while we obtained about 1.7 W/m/K. The snow media seem to be underestimated, at least by the diffusivity values. One can also see that all apparent values are walking, none of them is steady, which is to a common experience from other tests. At last, the soil media properties, shown on the Figure 8, illustrate the sense of the method. The normalization ensured that the Fourier number for the work and the reference media tests are equal in effect of normalization. The distinction of a respective sample and time, while evaluating properties, is sharp. It is not fuzzy, as it happens to defining a slope by a linear tangent, in presence of noise. When the requirement on equal Fo, is not fulfilled, then properties diverge slowly and steadily. When it is fulfilled, then the Fo formally picks up property values sharply. The apparent conductivity characteristics trace a loop–like path in the coordinate system. Soon after the start of heating, the diffusivity is highly overestimated, and the Fourier number is also overestimated. But after a few samples, Fo comes to about unity and starts varying steadily and uniformly. For a perfect wire, one should expect a property being nearly a constant. Apparent property values walk, while property values in absolute measure behave varying slowly slipping from one characteristic to another. This way, absolute property values go across a band of apparent characteristics.

Figure 9 shows that mechanism in detail. The case of a perfectly thin wire should correspond to a constant line plot, steadily representing a specific conductivity. For imperfect probe elements, a line splits on tilted characteristics and creates a need of selecting relevant values. Selecting a linear part of the temperature rise plot is an arbitrary choice, especially not bringing effects when the entire plot is non–linear at the beginning and delayed due to the heat capacity of sensing elements. Then the obtained values are shifted and walking with time. The presented procedures may be improved or corrected, but the use of the Fourier number seems to be fundamental.

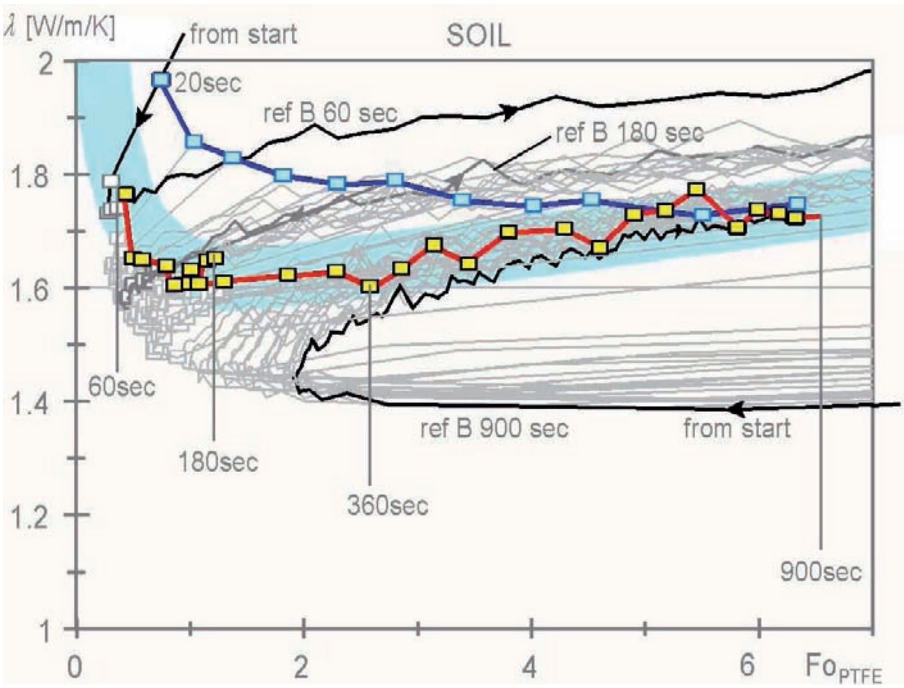

Figure 9: Selected bound of the apparent thermal conductivity characteristics (in grey), in absolute measure [W/m/K], for the test in soil media. Each characteristic corresponds to another choice of the reference time t_{refB}. Real and formally exact conductivity values, correspond only to the current reference time t_{refB}. Other values, are only apparent. A collective characteristic of real values (in red), goes across a bound of apparent characteristics. Nevertheless, the real measured values, still vary, and are spread in some tolerance band (shown in the background in pale blue). The highly concentrated minimal values (among the blocks in yellow), indicate the best choice of the time window, when the response shape does not deviate too much from media properties. The other characteristic of real values (in blue), is formed of the property values assigned to the third sample in the interval. The first point of this characteristic is denoted by the time=20 s. These samples indicated that determination of the measured property is quite reasonable even in 20 sec after switching the power heating on, and still improves with the larger reference time t_{refB}.

11 Conclusions

The way of interpreting properties from test data, made it clear that the cylindrical shape of a probe is not a critical condition for using the transient response method. Other shapes can be employed too. One can proceed the same or a similar way in absence of a linear slope in log-time measure. The sense of the method is to apply a heat impact and to derive a more or less precise estimate of the thermal properties of the medium. The Fourier number is a fundamental tool for that purpose. One interesting conclusion from this work is that the heating power value in absolute measure, was needed only for the calibration tests in reference media. For the work tests the power control was needed only in a relative measure, with respect to the calibration test.

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Appendices

11.1 Test equipment and test sites:

Figure 10: Tests and the test sites. The PTFE specimen with two probes under tests (top left). Two probes in snow under testing in open field conditions (bottom left). The test setup with water ice specimen (right).

Test data:

The test data listed below were obtained in two known media – PTFE, and ICE, and three media unknown by thermal properties – SOIL, SNOW1 and SNOW2, all with a use of the same MUPUS analog probe, and given for the same sensor No 7. That is believed that these data are sufficient to determine the unknown media thermal properties, without any additional detailed knowledge on the probe and any other knowledge on the unknown media. That is the subject of the paper. The properties obtained that way may be charged by some non-removable errors due to specific properties of the probe, but they should be mutually consistent. The data is the time series of the temperature $T[^oC]$, and the heating power q[mW/cm], for tests in particular media (PTFE, SOIL, water ICE, and snow: - SNOW1, and SNOW2).

time	PTFE		SOIL		ICE		SNOW2	(36.7%)	SNOW1	(19.1%)
sec	$[^oC]$	$[mW/cm]$	$[^oC]$	$[mW/cm]$	$[^oC]$	$[mW/cm]$	$[^oC]$	$[mW/cm]$	$[^oC]$	$[mW/cm]$
0[1]	22.479	275.317	-0.247	237.081	-46.603	321.500	-44.994	259.577	-44.756	301.723
10	26.836	272.949	3.598	235.170	-41.139	317.462	-38.688	255.836	-37.022	296.442
20	28.644	271.978	4.620	234.667	-40.027	316.653	-35.698	254.099	-33.457	294.069
30	29.803	271.359	5.166	234.399	-39.622	316.359	-33.531	252.855	-30.847	292.355
40	30.649	270.909	5.421	234.274	-39.460	316.242	-31.711	251.819	-28.719	290.973
50	31.368	270.528	5.631	234.172	-39.332	316.149	-30.459	251.112	-27.007	289.871
60	31.889	270.253	5.723	234.126	-39.217	316.066	-29.323	250.474	-25.570	288.952
70	32.353	270.008	5.921	234.030	-39.159	316.024	-28.408	249.962	-24.309	288.150
80	32.712	269.819	5.967	234.007	-39.159	316.024	-27.643	249.535	-23.223	287.463
90	33.094	269.617	6.095	233.945	-39.089	315.974	-27.017	249.188	-22.312	286.890
100	33.384	269.465	6.107	233.939	-39.101	315.982	-26.414	248.854	-21.490	286.373
110	33.628	269.337	6.176	233.905	-39.020	315.923	-25.927	248.585	-20.689	285.873
120	33.859	269.216	6.269	233.860	-38.985	315.898	-25.463	248.329	-20.009	285.449
130	34.056	269.112	6.293	233.848	-38.974	315.890	-25.116	248.138	-19.395	285.068
140	34.172	269.052	6.304	233.843	-38.927	315.856	-24.652	247.883	-18.835	284.721
150	34.427	268.918	6.362	233.814	-38.927	315.856	-24.444	247.768	-18.309	284.395
160	34.589	268.834	6.327	233.831	-38.916	315.848	-24.119	247.590	-17.815	284.091
170	34.740	268.755	6.351	233.820	-38.892	315.831	-23.945	247.495	-17.398	283.834
180	34.891	268.676	6.327	233.831	-38.916	315.848	-23.679	247.349	-17.003	283.591
190	34.972	268.634	6.455	233.769	-38.904	315.840	-23.493	247.248	-16.619	283.356
200	35.076	268.579	6.467	233.763	-38.869	315.815	-23.284	247.134	-16.268	283.141
210	35.238	268.495	6.490	233.752	-38.869	315.815	-23.169	247.071	-15.917	282.926
220	35.366	268.428	6.455	233.769	-38.835	315.790	-23.029	246.995	-15.588	282.725
230	35.470	268.374	6.490	233.752	-38.835	315.790	-22.844	246.894	-15.314	282.558
240	35.563	268.326	6.537	233.730	-38.811	315.773	-22.740	246.837	-15.018	282.377
250	35.667	268.271	6.513	233.741	-38.811	315.773	-22.612	246.768	-14.755	282.217
260	35.725	268.241	6.479	233.758	-38.777	315.748	-22.496	246.705	-14.513	282.071
270	35.876	268.163	6.571	233.713	-38.777	315.748	-22.276	246.585	-14.305	281.944
280	35.888	268.157	6.583	233.707	-38.788	315.756	-22.253	246.573	-14.074	281.804
290	36.015	268.090	6.583	233.707	-38.754	315.731	-22.160	246.522	-13.899	281.698
300	36.050	268.072	6.583	233.707	-38.730	315.714	-22.067	246.472	-13.636	281.538
310	36.154	268.018	6.630	233.684	-38.719	315.706	-21.986	246.428	-13.471	281.439
320	36.270	267.958	6.618	233.690	-38.754	315.731	-21.905	246.384	-13.274	281.320
330	36.293	267.946	6.537	233.730	-38.730	315.714	-21.812	246.334	-13.087	281.207
340	36.351	267.916	6.479	233.758	-38.754	315.731	-21.766	246.308	-12.923	281.108
350	36.397	267.892	6.595	233.701	-38.719	315.706	-21.697	246.271	-12.780	281.022
360	36.479	267.850	6.699	233.650	-38.719	315.706	-21.639	246.239	-12.626	280.929

Note 1: The data for the sensor no 7, within the sensor lenght L=1.60cm, the sensor inner radius a=0.39cm, the tube outer radius r=0.50cm, the tube wall thickness 0.10cm

Note 2: Due to non existing zero value in the log-time domain, the first sample time, is assigned not to 0sec, but to arbitrarily taken 0.1sec. Due to non existing zero value in the log-time domain, the first sample time, is assigned not to 0sec, but to arbitrarily taken 0.1sec. The heating interval was 900sec,but only 360sec range is presented as sufficiently long for determining thermal poperties

Table 5 provides the sensor length series, for the TP probe, in the order from top to bottom. This is a general information, showing possible capability of the probe for profiling. That information is not necessary for determining thermal properties on the base of Table 5, because the power related columns account power density per unit length, respecting the sensor length, given below.

Table 5: The sensor length series, in the probe

no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
L[cm]	0.91	1.00	1.09	1.20	1.32	1.45	1.60	1.77	1.95	2.16	2.38	2.64	2.92	3.24	3.59	3.99

PTFE thermal properties

According to NIST (`http://cryogenics.nist.gov/NewFiles/Teflon.html`), thermal properties of PTFE for the conductivity λ [W/m/K], and for the specific heat c_h [J/kg/K], are given by the following common expression:

$$\log x = a + b(\log T)^1 + c(\log T)^2 + d(\log T)^3 + e(\log T)^4 + f(\log T)^5 + g(\log T)^6 + h(\log T)^7 + i(\log T)^8$$

where

Table 6: Coefficients determining PTFE Thermal Properities

coeff.	for λ	for c_h
a	2.738	31.8825
b	-30.677	-166.5190
c	89.430	352.0190
d	-136.990	-393.4420
e	124.690	259.9810
f	-69.556	-104.6140
g	23.320	24.9927
h	-4.3135	-3.20792
i	0.33829	0.165032

The expression is applicable in the range of temperature T 4-300K, with accuracy not worse than 1.5%, given for the specific heat. The control values for T =300K, are following: λ=0.2728 [W/m/K], c_h=1115.3868 [J/kg/K] Bulk density of PTFE, was taken for this paper by a measurement of the specimen. The specimen was a cylinder 180mm long, the outer diameter was 100mm, with the central hole drilled axially for the inside diameter 10 mm. The bulk density was estimated as ρ_b=2200 [kg/m^3].

Water Ice Thermal Properties

There are three formulas on the thermal conductivity λ [W/m/K] of water ice, for the hexagonal kind:

11.1.1 Water Ice Model 1

The model is given by Steiner and Kömle (Steiner et al., 1991) in the formula
λ [W/m/K]=567/T,
where T is the temperature in [K].

Water Ice Model 2

The model is provided by
`http://www.es.ucl.ac.uk/research/planetary/undergraduate/dom/ices/ices.htm`
in the formula
$\log(\lambda)=M0+M1\cdot\log(T)$,
where the coefficients $M0$ and $M1$ are given in the following table.

Table 7: Water Ice Thermal Properities

Ice Type	Temperature range	Pressure [MPa]	M0	M1
Ice Ih	80-273K	20	2.7154	-0.9752
Ice Ic	150-200K	20	2.281	-0.813
Ice II	120-240K	240	2.842	-1.097
Ice III	180-250K	240	1.961	-0.822
Ice V	240-270K	530	1.56	-0.612
Ice VI	240-300K	1200	2.473	-0.928
Ice VII	275-300K	2400	2.491	-0.821
Ice VIII	240-370K	2400	4.193	-1.417
Amorphous ice	68-125K	100	-0.6496	0.2165

Water Ice Model 3

The model provided by

`http://www.astrobiology.nl/projects/planetary_ices.html#refs)`

is given in the formula for
λ [W/m/K]=656.3/T[K]-0.00077*T[K]
The models 1, 2, 3 do not provide bulk density and specific heat data.
These data were taken from another source, called the model 4, given below.

11.1.2 Water Ice Model 4

The engineering model for determining the conductivity, bulk density and specific heat vs temperature, for hexagonal ice, is given by the table, provided by

`http://www.engineeringtoolbox.com/ice-thermal-properties-d576.html)`

Table 8: Water ice thermal properties, engineering data

T [^{o}C]	ρ_b [kg/m^3]	λ [W/m/K]	c_h [kJ/kg/K]
0	916.20	2.220	2.050
-5	917.50	2.250	2.027
-10	918.90	2.300	2.000
-15	919.40	2.340	1.972
-20	919.40	2.390	1.943
-30	920.00	2.500	1.882
-40	920.80	2.630	1.818
-50	921.60	2.760	1.751
-60	922.40	2.900	1.681
-70	923.30	3.050	1.609
-80	924.10	3.190	1.536
-90	924.90	3.340	1.463
-100	925.70	3.480	1.389

Table 8 contains the engineering data on water ice properties. The paper accounted thermal properties of ice based on the model 1, after Steiner et al. (1991), without judging which of the three models is closer to the real physical properties of ice.

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