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S. Schuh & G. Handler

Further progress on solar age calibration

G. Houdek [1] and D. O. Gough [1,2]

[1] Institute of Astronomy, Madingley Road, Cambridge CB3 0HA, UK
[2] Department of Applied Mathematics and Theoretical Physics, Cambridge CB3 0WA, UK

Abstract

We recalibrate a standard solar model seismologically to estimate the main-sequence age of the Sun. Our procedure differs from what we have done in the past by removing from the observed frequencies a crude representation of the effect of hydrogen ionization and the superadiabatic convective boundary layer. Our preliminary result is $t_\odot = 4.63 \pm 0.02$ Gy.

Individual Objects: Sun

Introduction

Seismological calibration of solar models to estimate the age of the Sun necessarily depends predominantly on the frequencies of the lowest-degree modes which penetrate into the energy-generating core where the greatest evolutionary change in the stratification occurs. Most commonly this is accomplished by fitting the asymptotic formula

$$\nu_{n,l} \sim \left[n + \frac{1}{2} l + \hat{\epsilon} - \sum_{k=1}^{K} \left(\sum_{j=0}^{k} A_{k,j} L^{2j} \right) \left(\frac{\nu_0}{\nu_{n,l}} \right)^{2k-1} \right] \nu_0 \,, \tag{1}$$

to observed high-order frequencies $\nu_{\mathrm{o}n,l}$ of order n and degree l to determine the coefficients ν_0, $\hat{\epsilon}$, and $A_{k,j}$; here $L = l + \frac{1}{2}$. The most $l-$sensitive terms, at each degree $2k$–1, namely $A_{k,k}$, are, on the whole, the most sensitive to core conditions, and the least sensitive to the structure of the envelope (cf. Houdek & Gough 2007b). Therefore it is one or more of these that are the best determinants of stellar age. Eq. (1) is valid only if $l \ll n$ and n is large, such that the spatial scale of variation of the equilibrium state is everywhere much greater than the inverse vertical wavenumber of the mode. But that condition is not actually satisfied in the Sun: there is small-scale variation associated with ionization of abundant elements and the near discontinuity in low derivatives of the density at the base of the convection zone, which we call acoustic glitches, and which add the components $\nu_{\mathrm{g}n,l}$ to $\nu_{n,l}$ that are in general oscillatory with respect to n. Ignoring these components introduces systematic errors into a straightforward fitting of Eq. (1) to $\nu_{\mathrm{o}n,l}$, errors that are evident in the undulatory age estimates as the limits of the frequency range adopted for the fitting are varied (Gough 2001). In an attempt to obviate these errors, Houdek & Gough (2007a, 2008) estimated the glitch components $\nu_{\mathrm{g}n,l}$ by fitting to second differences (with respect to n) of the observed frequencies an asymptotic formula designed to represent the base of the convection zone and the two ionization zones of helium. In reality there is also an upper-glitch component, produced by the ionization of hydrogen and the upper superadiabatic boundary layer of the convection zone, which appears to be difficult to model in a reliable manner. When fitting the second differences Houdek &

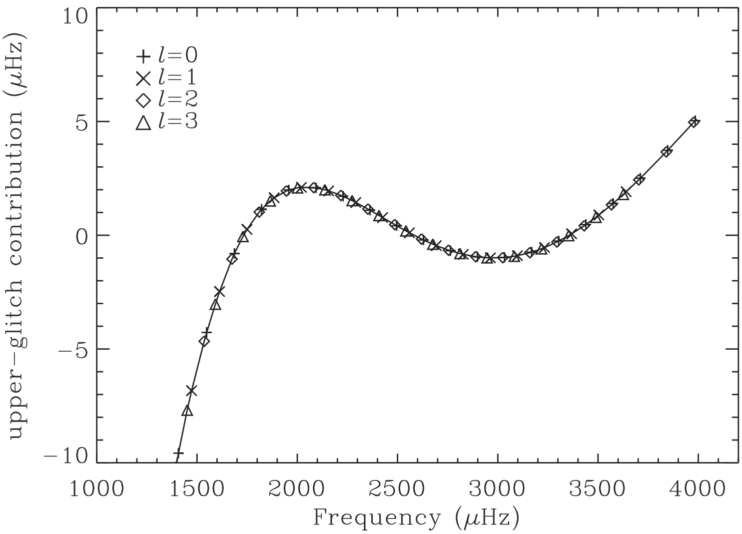

Figure 1: Contribution of the upper-glitch component to the glitch frequencies $\nu_{gn,l}$, obtained from the BiSON observations $\nu_{on,l}$ (Basu et al. 2007), as a function of $\nu_{on,l}$. It was obtained from summation of the series $P(\nu_{n,l})$, whose coefficients were determined from fitting to the second differences of the observed frequencies the second differences of an asymptotic formula representing the glitch components $\nu_{gn,l}$ (Houdek & Gough 2008). The upper-glitch component is produced by the ionization of hydrogen and the superadiabaticity of the surface boundary layer.

Gough (2007a) represented that component, coupled with the second differences of Eq. (1), somewhat arbitrarily as a series $P(\nu_{n,l})$ of inverse powers of $\nu_{n,l}$. Because the upper-glitch component is relatively smooth, they subsequently tacitly regarded it as being included in the smooth asymptotic expression (1) by adjusting the observed frequencies by only the component $\nu_{gn,l}$. Because the upper glitch is quite close to the surface (partly in the evanescent zones of the modes), its influence on the eigenfrequencies is essentially independent of l, and so should not have materially affected the fitted coefficients $A_{k,j}$ with $j > 0$.

Modification to the calibration procedure

In the work we report here we have tested the stability of the procedure by including in $\nu_{gn,l}$ a representation of the upper-glitch component. To this end we summed the second-difference representation P to obtain an estimate of its contribution to the frequencies. There is some ambiguity in how one separates smooth and glitch components near the surface, which is exhibited by the two undetermined constants of summation of the second differences; here we chose those constants by minimizing the error-weighted sum of the squares of the upper-glitch frequencies. The outcome is plotted in Fig. 1 using BiSON data (e.g. Basu et al. 2007) up to degree $l = 3$.

After fitting Eq. (1), with $K = 3$, to the resulting glitch-adjusted observed frequencies, the coefficients ν_0, $\hat{\epsilon}$ and $A_{k,0}$ were found to be naturally somewhat different from the results obtained without the upper-glitch adjustment. But the coefficients $A_{k,k}$ are similar. There is, however, a slight difference, which is evidently a product of an inadequacy of the asymptotic formulae to reproduce precisely the observed frequencies of the Sun.

Result

The result of the present model calibration against BiSON data (e.g. Basu et al. 2007) is

$$t_{\odot} = 4.63 \pm 0.02\,\mathrm{Gy}\,, \qquad (2)$$

a value in fair accord with our previous estimates (Houdek & Gough 2007b, 2008). The errors quoted here come solely from the stated observed frequency errors, which we have assumed to be statistically independent, and take no account of (systematic) errors in our procedure; that the value (2) differs from our previous estimates by as much as $2.5\,\sigma$ suggests that such systematic errors could be present at a level at least as great as the random errors. Our current value for the solar age is lower than the previous estimates by essentially this method, although, in contrast to many earlier estimates, it remains greater than the age of Model S of Christensen-Dalsgaard et al. (1996), which we used as our reference. It is also greater than that of many, if not all, meteorites. We have not yet completed our investigation of the robustness of the result, so we offer it still as a preliminary estimate.

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