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Food Security, Fertility Differentials and Land Degradation in Sub-Saharan Africa: A Dynamic Framework

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Abstract

We study the impact of differential fertility levels for the food-insecure and food-secure population on the long-run values of the population distribution and resources in a descriptive model, where the food security states are determined by a historically given food distribution and by the endogenous food production with resources and labor as inputs. Furthermore, we assume that the resource stock is reduced by poverty-driven environmental degradation. Moreover, we incorporate nutritional effects on labor productivity and mortality. By applying local bifurcation theory, we show that the model may exhibit multiple equilibria. Furthermore, the orbits of resources and the population distribution may be characterized by quasi-periodic behavior. Sustainable development in terms of approaching a steady state with positive values of resources and food-secure population is only promoted by low fertility levels of the food-insecure and food-secure population.

JEL Classification: C62, I32, Q01.

Keywords: Food Distribution, Fertility, Land Degradation, Dynamic Systems, Multiple Equilibria, Local Bifurcation.

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1 Introduction

"Over the past thirty years, most of sub-Saharan Africa has experienced very rapid population growth, sluggish agricultural growth, and severe environmental degradation" and there is evidence that these three phenomena are connected in a mutually reinforcing manner (Cleaver and Schreiber 1994). This nexus is commonly known as 'vicious circle' in the literature.

The concept of the 'vicious circle of poverty' dates back to the 1950s, when Ragnar Nurkse observed that "in discussion of the problem of economic development a phrase that crops up frequently is the 'vicious circle of poverty'. [...] It implies a circular constellation of forces tending to act and react upon one another in such a way as to keep a poor country in a state of poverty (Ascher and Healy 1990). Similarly, Lutz and Scherbov (2000) describe the vicious circle model as being "based on the assumption that high fertility, poverty, low education and low status of women and children are bound up in a web of interactions with environmental degradation and declining food production, in such a way that stress from one of these sources can trap certain rural societies, especially those living in marginal lands, into a vicious circle of increasingly destructive responses." Ascher and Healy (1990) tried to categorize this complex web of intervening factors into four distinct sectors: economic production, income distribution, natural resources and environment.[1] They argue that in this rather complex web, various vicious circles involving two or more sectors may be observed (for examples see Ascher and Healy 1990, pages 20–23).

One example of such a vicious circle is illustrated by the parable of firewood (Nerlove and Meyer 1997). Gathering firewood, water, etc. is mainly the task of women and children. As deforestation proceeds the borders of the forests are pushed further away from the village, then firewood collection becomes more time consuming. Consequently, children become more valuable for their parents and the demand for them may rise, which reinforces fertility. Summing up, environmental degradation may itself increase fertility and subsequently population growth.[2] Hence, Nerlove and Meyer (1997) argue that "[t]he relation between fertility and environmental degradation depends primarily on the way parents perceive the benefits of having children and not primarily on the effect of population size on the environment, as long as the environment is adversely affected by larger population". Similarly, Dasgupta (1993) finds that in poor countries children are also considered as income-earning assets which provides an additional motivation of households for having children.

In the spirit of the concept of the vicious circle, Lutz and Scherbov (2000) developed a quantitative simulation model, called PEDA (Population, Environment, Development and Agriculture), which links "population parameters and education to land degradation, food production and distribution" (Lutz and Scherbov 2000) and has been applied to several African countries, e.g. Burkina Faso, Cameroon, Madagascar, Mali, Uganda and Zambia (Lutz et al. 2002).

The PEDA model is a population-based simulation model by considering, besides age and sex structure, the literacy and food-security status as well as the place of residence of the

[1] However, Dworak (2002) argues that this categorization is still incomplete, since e.g. population (including its age and sex distribution, fertility, mortality, migration, education, etc.) and other institutional settings (security issues, property rights) are missing.

[2] This controversial causal link has been tested empirically for data of Pakistan and Nepal by Filmer and Pritchett (1997) and Loughran and Pritchett (1997) in Asia. Filmer and Pritchett (1997) verify several effects that enhance the plausibility of the hypothesis that environmental scarcity could possibly raise the demand for children. Although, there are several findings which may be inconsistent with the hypothesis. However, Loughran and Pritchett (1997) reject the hypothesis, i.e. environmental scarcity lowers the demand for children for Nepalese data. For sub-Saharan African cross-country data Cleaver and Schreiber (1994) find a positive relation between total fertility rates and the rate of deforestation. However, the significance was ambiguous (Cleaver and Schreiber 1994).

population. In sum, the PEDA model refers to about 1600 population states. It considers a natural resources module (land and water) and an agricultural production module and contains a food distribution mechanism, in order to account for the inequality in the access to food.

In particular, the PEDA model includes the vicious circle reasoning. The growth of the rural (illiterate and food-insecure) population contributes to land degradation, thus lowering agricultural production, and thereby increasing the number of food-insecure population. However, the PEDA model does not assume increasing fertility as a direct response to food insecurity because of its controversial empirical foundation. "Rather, the food-secure and food-insecure fractions of the population are assumed to have different fertility levels (subject to exogenously-defined trends) and hence the aggregate fertility level only responds to changes in the food insecurity through changing weights of the groups in the calculation of overall fertility." (Lutz et al. 2003)

In this paper, we aim to investigate the assumption of differential fertility levels for the food-insecure and food-secure population in a reduced-form derivative of the PEDA model as presented in Lutz et al. (2002). Furthermore, we extend the PEDA framework by incorporating nutritional effects on labor productivity and mortality. In short, each time-period, the rural food-secure and food-insecure population together with the natural resource stock determine total food production. But, the population lowers the resource stock by land degradation. The total amount of food produced is distributed each time-period according to the prevailing food distribution function, thereby dividing the population into food-secure and food-insecure population.

The remainder of the paper is organized as follows. Section 2 presents the model and discusses the underlying assumptions. Section 3 investigates the long-run evolution of the resources, the food-insecure and food-secure population. In particular, by applying bifurcation theory we analyze the effects of changing degree of inequality and fertility differentials.[3] Finally, section 4 concludes and presents further extensions.

2 The Model

Population Dynamics

At each time point t, population P_t consists of two groups, the food-secure population, denoted by P_{St}, and the food-insecure population, P_{It}, with $P_t = P_{St} + P_{It}$. Let us denote the threshold level $\tilde{y}$ as the minimum level of calories necessary to be food secure. Then, a person belongs to group P_{St} or P_{It}, respectively, if he or she receives a level of food that exceeds (falls short of) $\tilde{y}$. The dynamics of the sub-populations are composed of natural population growth and the transition between the food security states (cf. Figure 1), i.e.[4]

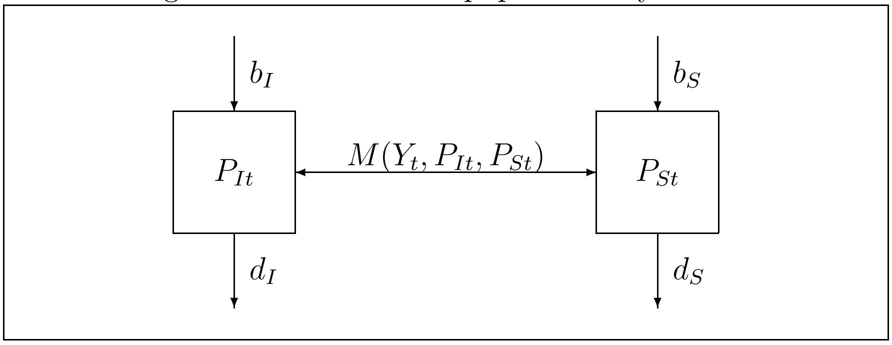
$$\Delta P_{It} = (b_I - d(y_{It}))P_{It} + M(Y_t, P_{It}, P_{St}) \tag{1}$$

$$\Delta P_{St} = (b_S - d(y_{St}))P_{St} - M(Y_t, P_{It}, P_{St}), \tag{2}$$

[3]In a substantially different but methodological related context, Kremer and Chen (1999) show that the positive feedback between fertility differentials and income inequality may lead to multiple steady states of the long-run population distribution of skilled and unskilled population. In particular, Kremer and Chen (1999) investigate the positive feedback between fertility differential and income inequality created by the fact that higher wages reduce fertility and that children of the unskilled are more likely to be unskilled than of the skilled.

[4]The symbol Δ represents the first order difference, i.e. $\Delta x_t = x_{t+1} - x_t$. Furthermore, the prime after function names denotes the first derivative and the double prime the second derivative.

Figure 1: Flow chart of population dynamics.



where b_I and b_S denote the (exogenously given) birth rate for the food-insecure and food-secure population, respectively. We assume, that the death rates of the food-insecure and food-secure population depend on the per capita food entitlement of the respective sub-population, y_{It} and y_{St}, where mortality decreases when food entitlement rises, i.e. $d' < 0$.

Obviously, the transition from food-secure to food-insecure population represented by the function M hinges on food Y_t to be distributed among the two sub-populations P_{It} and P_{St} and on the distribution of food.

In particular, we assume the following functional form for the crude death rates

$$d(y_j) = d_{nat} + \frac{d_{max}}{1 + \exp\left(\delta(y_j - y_m)\right)}, \qquad \text{for } j = I, S, \tag{3}$$

which has been proposed in Strulik (1995) and has also been used in Prskawetz et al. (2000). In particular, d_{nat} indicates a minimum death rate which is approached for values of the per capita food entitlement rising to infinity. Furthermore, $d_{nat} + d_{max}$ are approximately the maximum death rate for a per capita food entitlement close to zero. Within these upper and lower bounds of mortality, the crude death rates fall with increasing per capita food entitlement, where the rate of decrease is initially rising and then decreasing. Furthermore, δ measures the slope of the mortality function. Finally, if the per capita food entitlement equals y_m, then the mortality amounts to about half of its maximum value (Strulik 1995).

Moreover, it seems reasonable to assume that $b_I, b_S < d_{nat}+d_{max}$ holds, which implies that for a per capita food entitlement close to zero, the natural growth rates of the sub-populations are negative.

Food Distribution

At the end of each time period the total amount of food available Y_t is distributed among the population. We postulate a historically given food distribution function which is represented by a Lorenz curve in order to be consistent with the original PEDA model (for alternative measures of inequality like the variance, the Gini coefficient, etc. see Atkinson 1970, Rothschild and Stiglitz 1973). This curve plots cumulative shares of food $L(F(z))$ as a function of cumulative population shares $F(z)$ when individuals are ranked in increasing order of the food z that they receive (see Appendix A.1 for a discussion of the Lorenz curve). For instance, $L(P_{It}/P_t)$ indicates the share of total food which the food-insecure population receives.[5] The food distribution function together with the total amount of food Y_t to be distributed in each

[5]Hence, the per capita food entitlement at time t of the food-insecure population is defined as $y_{It} = Y_t L(P_{It}/P_t)/P_{It}$. Furthermore, the per capita food entitlement at time t of the food-secure population is given by $y_{St} = Y_t(1 - L(1 - P_{St}/P_t))/P_{St}$.

time period and the threshold level of food $\tilde{y}$ determine the share of food-secure $P_{S,t+1}$ and food-insecure people $P_{I,t+1}$, respectively, in the following period.

Recalling some basic mathematical properties of the Lorenz curve, we can derive an analytical expression for the transition from the food-secure population to the food-insecure population P_{It} in each time period.[6] We exploit the fact that the slope of the Lorenz curve at any point $F(z)$ is inversely proportional to per capita food production $y = Y/P$ and proportional to the corresponding food level z (see Appendix A.1 for the derivation of this result)

$$\frac{dL(F(z))}{dF(z)} = l(F(z)) = \frac{1}{y}z. \tag{4}$$

Assuming further that we can analytically solve for the inverse of the derivative of the Lorenz curve, equation (4) can be written as

$$F(z) = l^{(-1)}\left(\frac{z}{y}\right). \tag{5}$$

Since F represents a distribution function, it has to be constrained by one from above. Obviously, the corresponding level of food z_{max} for which $F(z_{max}) = 1$ holds first, indicates the maximum level of food entitlement in the economy. Equation (5) then implies that the maximum level of food will depend on the prevailing level of per capita food production y and the functional form of the Lorenz curve. Unless the economy is in a stationary state, per capita food production y and henceforth the maximum level of food z_{max} will vary over time.

Recalling that $\tilde{y}$ indicates the threshold level of food, a person needs to be food secure, equation (5) evaluated at $z = \tilde{y}$ gives the proportion of the population becoming food insecure. Hence, if $F(\tilde{y}, y)$ exceeds the actual proportion of food-insecure population P_{It}/P_t, then the stock of food-insecure people will rise.[7] Consequently, the movement from food-secure population to food-insecure population is determined by

$$M(Y_t, P_{It}, P_{St}) = \left(F(\tilde{y}, y_t) - \frac{P_{It}}{P_{It} + P_{St}}\right)(P_{It} + P_{St}), \tag{6}$$

where $y_t = Y_t/(P_{It} + P_{St})$. Note, that when $\tilde{y}$ exceeds the maximum food entitlement z_{max}, then $F(\tilde{y}, y)$ equals one and the entire population will become food insecure in the next period.

Figure 2 illustrates the derivation of the share of the population that falls short of the minimum requirement graphically. According to condition (4), the slope of the Lorenz curve evaluated at the share of the population falling short of the minimum requirement equals the ratio of the minimum requirement and the per capita food entitlement. Hence, for a given minimum requirement and per capita food entitlement it remains to find the abscissa value, where the condition (4) holds, as illustrated in Figure 2 for two examples (points A and B). Obviously, the higher the per capita food entitlement relative to the subsistence level is, the less is the share of the population falling short of the latter.

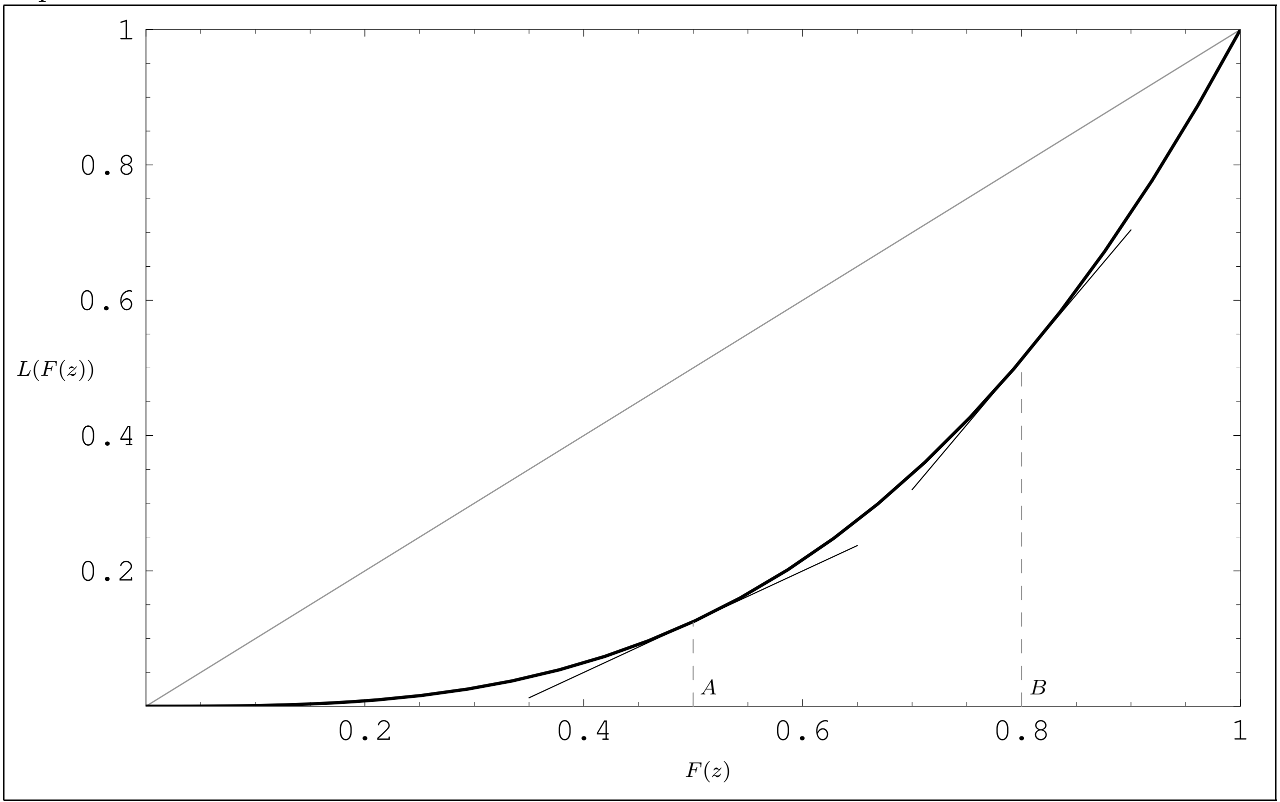
Food is produced according to a Cobb-Douglas production function with the inputs agricultural labor, represented by the rural population, and resources.[8] Aggregation over the

[6] To simplify the notation we shall skip the time argument in the subsequent mathematical derivations.

[7] In order to highlight the dependency of the share of the population falling short of the minimum requirement $F(\tilde{y})$ on per capita food entitlement, we add y as second argument in what follows.

[8] In this analysis, we restrict to food crops and abstain from cash crops. Hence, markets are not modelled.

Figure 2: Graphical derivation of the share of the population falling short of the minimum requirement of calories.



exogenous production factors and production adjustments of the PEDA model, as outlined above, yields the production function

$$Y_t = T(h_I P_{It} + h_S P_{St})^{\beta_1} R_t^{\beta_2}, \quad h_I + h_S = 1, \tag{7}$$

where h_I and h_S denote constant values of labor efficiency of the food-insecure and food-secure population, respectively. Furthermore, we normalize the efficiency units and we assume that $h_I < h_S$. The latter assumption is based on the fact that "a person's consumption-intake affects his productivity" (Dasgupta and Ray 1987).[9] Furthermore, T indicates the constant technology parameter and R_t represents the resource stock at time t. In addition, β_1 and β_2 denote the production elasticities with respect to labor and resources, respectively.

For the numerical analysis we postulate the following simple functional form for the Lorenz curve[10]:

$$L(F(z)) = (F(z))^{\alpha} \qquad \alpha > 1. \tag{8}$$

This simple Lorenz curve fulfills the assumption that the first derivative is analytically invertible, which has been postulated above. Moreover, it has the advantage that a single parameter, namely α, uniquely governs the degree of inequality and therefore ranks the food distributions.

However, the specific functional form (8) for the Lorenz curve implicitly assumes a zero minimum food entitlement. Since the minimum requirement of calories in order to be food secure is strictly positive, there will always be a strictly positive number of people falling

[9] An empirical confirmation of the nutrition-productivity hypothesis using household-level data from Sierra Leone is provided in Strauss (1986).

[10] See Chotikapanich (1993) for alternative functional forms of the Lorenz curve.

short of this minimum requirement. Hence, we will not obtain a steady state with a zero number of food-insecure population.

By using this specific form of the Lorenz curve, equations (5) and (6) are transformed to

$$F(z) = \left(\frac{z}{\alpha y}\right)^{1/(\alpha-1)}, \tag{9}$$

and

$$M(Y_t, P_{It}, P_{St}) = \left(\min\left\{\left(\frac{\tilde{y}}{\alpha y}\right)^{1/(\alpha-1)}, 1\right\} - \frac{P_{It}}{P_{It}+P_{St}}\right)(P_{It}+P_{St}), \tag{10}$$

where $y_t = Y_t/(P_{It}+P_{St})$.

Resource Dynamics

The stock of resource increases by indigenous growth/regeneration and is lowered by environmental degradation, i.e.

$$\Delta R_t = g(R_t) - D(P_{It}, P_t, R_t), \tag{11}$$

where we assume a declining rate of regeneration. That means, the higher the resource stock is, the lower is the rate of regeneration. Hence, we assume a saturation level or maximum level of the resource stock, $\bar{R}$, which is the stationary solution of R if the resource is not degraded. In particular, we assume that regeneration can be represented by the difference of the maximum level, $\bar{R}$, and the actual resource stock, R_t, times the speed at which the resource regenerates, a:

$$g(R_t) = a(\bar{R} - R_t)$$

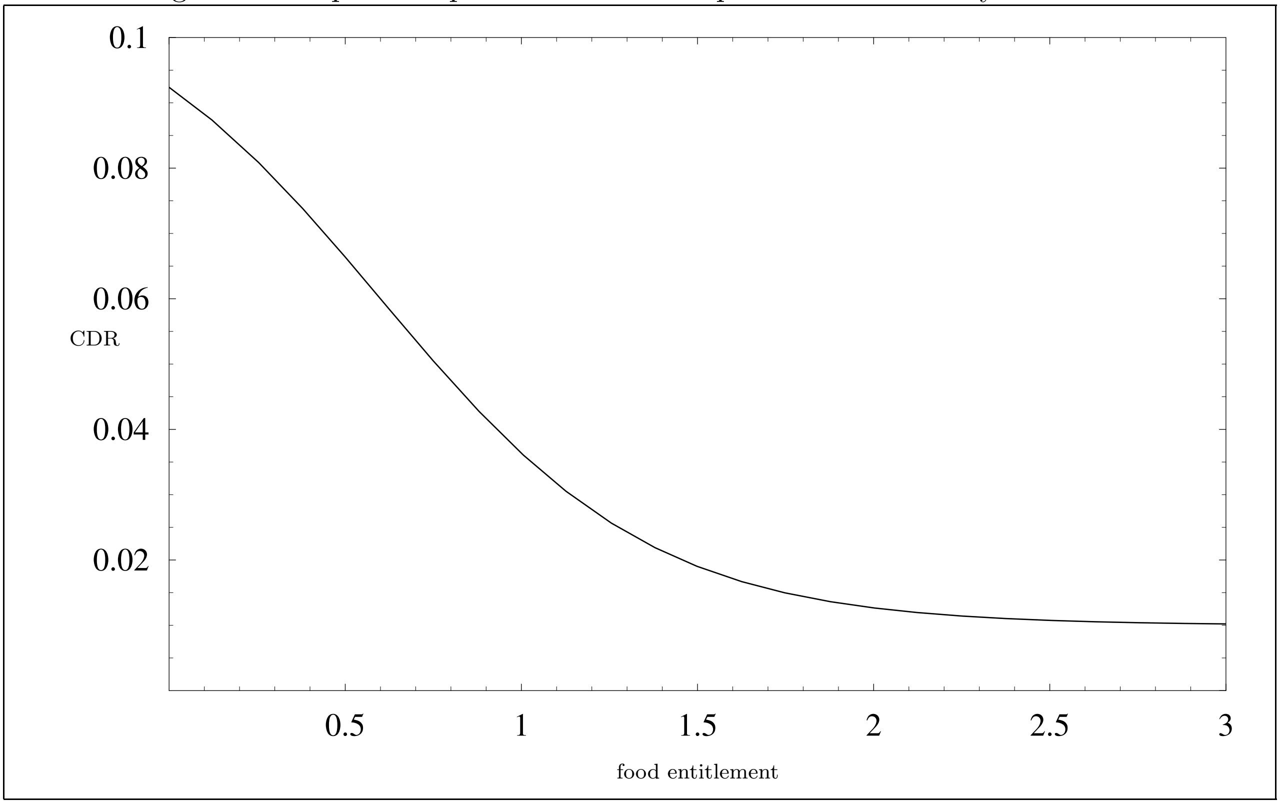
In addition, we assume that resource degradation hinges on the number of food-insecure population and on the stock of available resources. More specific, we postulated that the degradation function is increasing in both arguments. In particular, Lutz et al. (2002) assume

$$D(P_{It}, P_t, R_t) = \gamma \frac{P_t}{\bar{R}} P_{It} \frac{R_t}{R_t + \eta} \tag{12}$$

where γ and η are fixed parameters. While, it is assumed that degradation increases linearly in P_I, degradation is modelled to rise at a decreasing rate with the level of the resources. This assumption reflects the fact that environmental stress is less, the higher the available resource stock is. Furthermore, degradation is zero, if the resources are completely degraded. Similarly, if the entire population is food secure, the resources will not be degraded.[11] Moreover, degradation is scaled by population density in order to reflect the fact that poverty induced environmental degradation is reinforced by population pressure. Dworak (2002) conducts a sensitivity analysis of the results with respect to the functional form of the degradation function in a framework of zero population growth. Surprisingly, the results show to be rather robust with respect to the functional form of the resource equation. Furthermore, Dworak (2002, chapter 8) reformulates the resource equation in order to incorporate soil quality and quantity explicitly, as they are affected differently by population pressure.

[11] "Land degradation is by no means caused only by the poor. Irresponsible rich farmers sometimes exploit the land, but by and large, farmers with secure tenure and capital are more likely to conserve natural resources. When natural disasters occur they can turn to alternative sources of income, borrow and repay in better years. These alternatives are not open to the poor" (Young 1998). The focus of this paper is to analyze how the vicious circle of poverty and environmental degradation can be broken. Therefore, we only model poverty-driven environmental degradation and disregard differently motivated degradation.

Figure 3: Graphical representation of the postulated mortality function



Calibration

The parameter values used in the remainder of this paper are calibrated for the case of Mali. In particular, we use the parameters given in Lutz and Scherbov (2000), which were also used in Dworak (2002). However, the model studied here extends the PEDA framework by incorporating nutritional mortality effects. The calibration of the parameter values of the mortality function as well as the assumptions about fertility is described below.

The time series data necessary to estimate the parameters d_{nat}, d_{max}, and δ are sparse or hardly to obtain, if they even exist. For instance, the reports about the state of food insecurity (Food and Agriculture Organization 1999, 2000, 2001), which appear annually since 1999, estimate only in the year 2000 the depth of food insecurity for the single countries for 1996–98. Hence, there are only estimates for the per capita dietary energy supply of the food-insecure and food-secure population for 1996–98. Moreover, there are not any data about mortality by food security status. However, the latter constraint can be overcome by writing the crude death rate of the total population as weighted average of the crude death rates of the sub-populations, where the weights are given by the share of the respective sub-population, which are monitored annually by the Food and Agriculture Organization of the United Nations, i.e.

$$\begin{aligned} d_t &= d(y_{It})\frac{P_{It}}{P_t} + d(y_{St})\left(1 - \frac{P_{It}}{P_t}\right) \\ &= d_{nat} + d_{max}\left(\frac{P_{It}/P_t}{1+\exp\left(\delta(y_{It}-y_m)\right)} + \frac{1-P_{It}/P_t}{1+\exp\left(\delta(y_{St}-y_m)\right)}\right). \end{aligned} \tag{13}$$

Nevertheless, the data sets are too sparse in order to obtain reliable parameter estimates. Therefore, we have to base our parameter estimations primarily on assumptions than on real data. For instance, Strulik (1995) assumes that d_{nat} and d_{max} equal 0.01 and 0.1, respectively, which seem plausible also for our purpose. Furthermore, it seems reasonable to assume that y_m is less than the minimum requirement of calories in order to be food secure. For the following simulations we postulate that y_m equals 0.6, which is half of the minimum requirement in order to be food secure. Combining these assumptions with the estimates for the crude death rate of the total population, the share of the food-insecure population, and the per capita dietary energy supply for the food-insecure and food-secure population, respectively, yields an estimate for the slope of the mortality function, δ, about 2.57. Figure 3 plots the mortality function under the assumption of these parameter values.

Moreover, there does not exist any time series data of fertility by food security status. Hence, the fertility differential between the food-insecure and food-secure population has to be based on assumptions.

Finally, Table 1 summarizes the parameter values used.

Table 1: Summary of parameter values.

Parameter	**Value**	**Description**
a	0.02	Indigenous growth rate of resources
α	1.3	Degree of inequality
b_I	0.05	Crude birth rate of the food-insecure population
b_S	0.03	Crude birth rate of the food-secure population
β_1	0.534	Production elasticity with respect to labour
β_2	0.088	Production elasticity with respect to resources
d_{nat}	0.01	Crude death rate independent from food entitlement
d_{max}	0.1	Maximum crude death rate which depends on food entitlement
δ	2.57	Slope of the mortality function
η	$0.\dot{6}$	Concavity parameter of the degradation function
γ	$0.00\dot{8}$	Degradation impact factor
h_I	0.2	Labor efficiency of the food-insecure population
h_S	0.8	Labor efficiency of the food-secure population
$\bar{R}$	1	Maximum resource stock
T	2.28	Technology parameter of the production function
$\tilde{y}$	1.2	Minimum requirement of calories
y_m	0.6	Food entitlement, where the crude death rate reaches half of its maximum value

3 Dynamics and Bifurcation Analysis

Putting the pieces together, the dynamical system studied in this paper is given by

$$P_{I,t+1} = (1 + b_I - d(y_{It}))\, P_{It} + M(Y_t, P_{It}, P_{St}) \tag{14}$$

$$P_{S,t+1} = (1 + b_S - d(y_{St}))\, P_{St} - M(Y_t, P_{It}, P_{St}) \tag{15}$$

$$R_{t+1} = R_t + a(\bar{R} - R_t) - \gamma \frac{P}{\bar{R}} P_{It} \frac{R_t}{R_t + \eta}. \tag{16}$$

In the remainder of the paper, we proceed as following. First, we compute the steady states of the difference equation system (14)–(16). By applying local bifurcation theory, we determine regions of different long-run evolution of the orbits (i.e. multiple equilibria, quasi-periodic orbits) in the parameter space in order to identify the long-run effects of parameter changes.

Proposition 1 *If the natural population growth rate of the food-insecure population for a per capita food entitlement of $\tilde{y}/\alpha$ is positive, then there exists a non-trivial steady state where the entire population is food insecure.*

The proof is given in Appendix A.2. However, whether there exist further fixed points with an equilibrium share of the food-insecure population less than one can only be determined numerically.

In Figures 4–5, we investigate numerically the long-run behavior of the dynamical system (14)–(16) by jointly changing the degree of inequality and the fertility differential, as well as the crude birth rates, respectively. In particular, there exist either one or three fixed points. As evident from Figures 4–5, the orbits may become quasi-periodic in the long run as represented by a closed invariant curve in the state space.[12]

In particular, in Figure 4 the effect of a jointly changing degree of inequality and the fertility differential between the food-insecure and food-secure population is investigated, where we fix the crude birth rate of the food-insecure population. For α exceeding 1.584, there exists only one fixed point, which is stable (region ⓓ) or unstable and surrounded by a stable closed invariant curve (region ⓔ). In contrast for α less than 1.584, it is also possible that three fixed points exist (regions ⓑ and ⓒ), where the equilibrium number of food-secure population equals zero for one of the fixed points. Furthermore, the latter fixed point is always stable, where in region ⓑ there exists a second stable fixed point, which is unstable in region ⓒ. This second equilibrium exhibits a high equilibrium resource stock and a relatively low population size, where the equilibrium share of the food-insecure population is low.

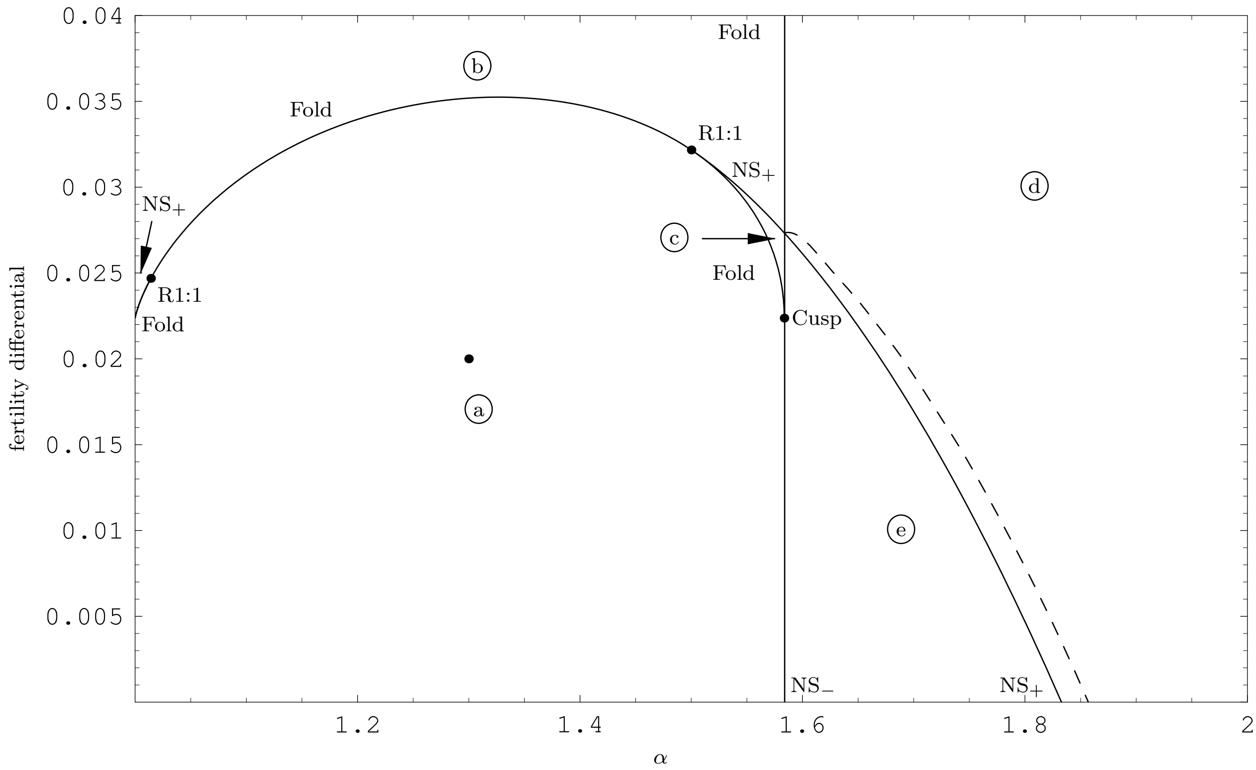
Moving from the left to the right in Figure 4 the distribution effect, which is extensively discussed in Prskawetz et al. (2003), can be detected. In particular, Prskawetz et al. (2003) investigate the effect of the inequality in the distribution of food in the PEDA framework under the assumption of zero population growth and constant resources. Thereby, they split the ambiguous effect of varying the degree of inequality in a distribution and a production effect. In particular, an increase in the degree of inequality, ceteris paribus, will intuitively increase the share of the food-insecure population as more food is distributed to the upper classes. However, Prskawetz et al. (2003) demonstrate that this must not necessarily be true. It may happen that the share of food-insecure population even decreases, as the degree of inequality increases.[13] In this case, food entitlement of the poorest of the poor is shifted to the upper food entitlement classes, thereby increasing also the food entitlement of those who just fall short of the minimum requirement

In addition, a change in the degree of inequality also effects the maximum food entitlement αy and henceforth determines whether the entire population will be food insecure in the long run. In particular, as more food is distributed to the upper food entitlement classes, the maximum food entitlement of the economy will rise (*distribution effect*). However, as the number of food-insecure population varies due to a change of the degree of inequality, the total amount of food produced and henceforth, the maximum food entitlement vary, too (*production effect*). If the share of the less efficient food-insecure people increases as a consequence of a rising degree of inequality, the total amount of food produced declines and

[12]For the technical details of the bifurcation analysis see Appendix A.3.

[13]Prskawetz et al. (2003) also demonstrate that the existence of such a result can be shown for a general food distribution function by using mean preserving spreads as introduced by Rothschild and Stiglitz (1970).

Figure 4: Bifurcation diagram with respect to the degree of inequality, α and the fertility differential between the food-insecure and food-secure population. All other parameters are set as in Table 1. The dot indicates the parameter values of α and the fertility differential, $b_I - b_S$ assumed in Table 1. For the technical details of the bifurcation analysis see Appendix A.3.



ⓐ Unique fixed point with $\hat{P}_S = 0$

ⓑ Three fixed points: two stable, where one exhibits $\hat{P}_S = 0$, and a saddle.

ⓒ Three fixed points: one stable, where one exhibits $\hat{P}_S = 0$, and two saddles.

ⓓ Unique stable fixed point, with $\hat{P}_S > 0$.

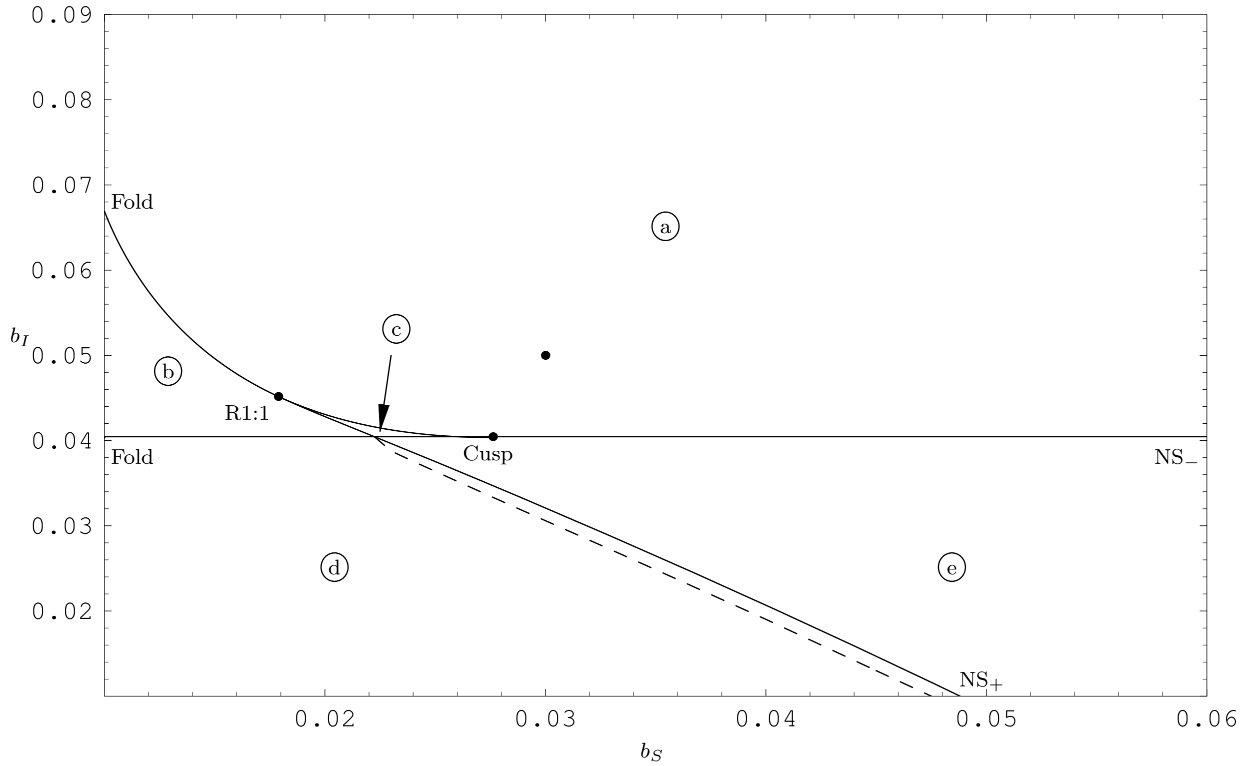
ⓔ Unique unstable fixed point surrounded by a stable closed invariant curve.

thus lowering the maximum food entitlement. Summing up, if a negative production effect dominates the distribution effect, the maximum food entitlement decreases and the probability that the entire population will be food insecure in the long run, increases. However, if the distribution effect dominates, then at least the richest person is entitled to sufficiently food to be food secure in the long run. Hence, the equilibrium share of the food-insecure population falls below one if the degree of inequality is sufficiently increased (cf. Figure 4 as α passes the value 1.584).

Moving in the left part of Figure 4 from the bottom to the top (i.e. from region ⓐ to ⓑ) a second stable equilibrium emerges. In this case, the fertility level of the food-secure population is sufficiently lower than that of the food-insecure population implying that the natural growth rate of the food-secure population may turn negative and there may be a positive number of food-secure people in the long run. However, the negative natural population growth rate for the food-secure population implies a continuous flow from the food-insecure to the food-secure population in the equilibrium. Hence, for the existence of such an equilibrium a high resource stock together with a low total population size is required.

However, the results presented in Figure 4 crucially depend not only on the fertility

Figure 5: Bifurcation diagram with respect to the crude birth rates of the food-insecure and food-secure population. All other parameters are set as in Table 1. The dot indicates the parameter values of b_S and b_I assumed in Table 1. For the technical details of the bifurcation analysis see Appendix A.3.



(a) Unique fixed point with $\hat{P}_S = 0$

(b) Three fixed points: two stable, where one exhibits $\hat{P}_S = 0$, and a saddle.

(c) Three fixed points: one stable, where one exhibits $\hat{P}_S = 0$, and two saddles.

(d) Unique stable fixed point, with $\hat{P}_S > 0$.

(e) Unique unstable fixed point surrounded by a stable closed invariant curve.

differential between the food-insecure and the food-secure population but also on the fertility level of the food-insecure population, which was held fixed so far. Hence, Figure 5 illustrates the change of the long-run behavior as the crude birth rates of the food-insecure and food-secure population are varied simultaneously. For high fertility of the food-insecure population, everybody will be food insecure in the long run. If the crude birth rate of the food-secure population is sufficiently low, a second fixed point with a low total population size but with a positive share of the food-secure population emerges. A low crude birth rate of the food-secure population in the presence of high fertility levels of the food-insecure population may decrease the share of the food-insecure population in the long run but simultaneously implies also a low total population in equilibrium. The long-run behavior drastically changes if the crude birth rate of the food-insecure population falls below 0.04. Then, there exists either a stable fixed point with a positive number of food-secure population or for higher fertility levels of the food-secure population there exists an unstable fixed point surrounded by a stable closed invariant curve as illustrated already in Dworak (2002, chapter 6). Consequently, low fertility levels of both sub-populations promote sustainable development. However, we did not consider crude birth rates below 0.01 in our analysis, since for such low fertility rates the population will be extinct in the long run. Proposition 2 summarizes the findings.

Proposition 2 *Low fertility levels of the food-secure population in the presence of high crude birth rates of the food-insecure population enhances chances for sustainable development by eventually decreasing the long-run share of the food-insecure population but implies simultaneously a low total population in equilibrium. Only low fertility levels of both sub-populations yield sustainable development for all initial values.*

4 Conclusions and Extensions

The major part of the literature concerning income distribution and differential fertility is devoted to the context of investment in human capital.

As noted in the introduction, Kremer and Chen (1999) show that the positive feedback between fertility differentials and income inequality may lead to multiple steady states of the long-run distribution of skilled and unskilled population. Translating the skilled and unskilled population into food-insecure and food-secure population, where labor productivity depends on the nutritional status, enables us to compare our results to Kremer and Chen (1999).

Nevertheless, the model of Kremer and Chen (1999) differs substantially from our framework in several aspects. First, Kremer and Chen (1999) derive multiple steady states of the ratio of unskilled to skilled population, where the population growth rates are constant, but not necessarily equal to zero, in the steady states. Hence, Kremer and Chen (1999) consider a stable population rather than a stationary population in the equilibrium. Since we are considering a framework of limited resources and constant technology, the population has to become stationary in order to obtain a steady state.

Secondly, Kremer and Chen (1999) derive an inverse relationship between fertility rates and wages by assuming a simple utility function. Hence, they endogenously define the population growth rates. In contrast, PEDA assumes different, but constant fertility levels for the food-insecure and food-secure population, respectively. Therefore, overall fertility rates vary only through compositional changes of the population.

However, Kremer and Chen (1999) and the PEDA framework have in common that they impose an exogenously given distribution of education costs in the case of Kremer and Chen (1999) and of food in the PEDA model. This exogenously defined distribution mainly governs the dynamics of the long-run population distribution.

In contrast, Galor and Zeira (1993) explore the relationship between wealth distribution and investment in human capital in an overlapping generations framework with intergenerational altruism, where the wealth distribution endogenously evolves over time via bequests. They also find that the long-run distribution of skilled and unskilled workers depend on the initial wealth distribution.[14] However, Galor and Zeira (1993) derive multiple equilibria of the long-run distribution of the skilled and unskilled population in a framework of zero population growth. Similarly, Dworak (2002, chapter 5) finds multiple steady states of the long-run distribution of the food-insecure and food-secure population in the reduced-form derivative of the PEDA model under the assumption of zero population growth for both sub-populations. Hence in Galor and Zeira (1993) and Dworak (2002, chapter 5), multiple equilibria are mainly generated by the positive feedback between the income/food distribution and labor productivity depending on the educational/nutritional status within a framework of stationary populations.

[14] The existence of the multiple steady states of the long-run distribution basically depends on the assumption of technological non-convexity and imperfect credit markets (Galor and Zeira 1993).

Maintaining the assumption of differential population growth for the food-insecure and food-secure population but setting aggregate population growth to zero, Dworak (2002, chapter 6) identifies that differential population growth only affects the domain of attraction of the respective steady states. Adding aggregate, exogenous population growth reduces the number of equilibria to a unique steady state. However, this unique steady state may be unstable and the orbits either tend to unlimited growth of the population while resources become extinct or the total population size approaches zero while the resources tend to its maximum level in the long run (Dworak 2002, chapter 6). Assuming endogenous death rates prevents that the population grows unlimited in the long run and henceforth, allowing multiple equilibria again.

The aim of the paper was to identify the role of differential fertility of the food-insecure and food-secure population within a framework of unequal access to food. We find out that a higher fertility differential in terms of the difference of the fertility level of the food-insecure population minus that of the food-secure population may decrease the steady state number of food-insecure population, but the latter is mainly driven by the degree of inequality and the respective fertility levels. In particular, only low fertility levels for both sub-populations promote sustainable development in terms of positive levels of resources and population and independent of the initial distribution of food-secure versus food-insecure population.

These results highlight the role of population growth as it is linked to poverty. As we demonstrate, food insecurity may even persist in a stationary population with an unequal distribution of food, where population growth exacerbates poverty. In our framework, fertility reductions of the food-insecure and food-secure population mitigate the stress on resources and subsequently on food production. The latter result is a prominent claim of classical economics. However, it is even acknowledged by some of the neoclassical economists, who regard high population growth as a neutral factor, "that fertility reduction can buy time while resource substitutes are found or market or institutional inefficiencies are addressed" (Jolly 1994).

In the PEDA framework, the institutional settings are only reflected by the assumption of a historically given, unequal food distribution. However, there is no justification given about the underlying assumptions about the institutions (e.g. property regimes, access to markets), although the institutional settings reflect the transition between the food-security states as well as the resource equation, particularly, the degradation function. Beaumont and Walker (1996) nicely address the latter, where they investigate the optimal choice of labor input and intensity of farming under different property regimes. Dworak (2002, chapter 8) proposes a reformulation of the resource equation by distinguishing between quantity and quality of soil as they are reflected differently by population pressure. A combination of these two approaches would substantially improve the modelling of the resource side in the PEDA framework.

Another drawback of the PEDA framework is that the degree of inequality is held constant over time. Galor and Zeira (1993) present an endogenously evolving wealth distribution by the means of micro-foundation of bequests. The incorporation of bequests together with the property regimes would make the PEDA framework much more realistic.

A Appendix

A.1 Discussion of the Lorenz Curve

As already stated above, the Lorenz curve $L(F(z))$ plots cumulative shares of income as a function of cumulative population shares $F(z)$ when individuals are ranked in increasing order of income z (Lam 1988, p. 143 ff).

Consequently, the Lorenz curve is a monotonically increasing and convex function, where $L(0) = 0$ and $L(1) = 1$ (see Chotikapanich 1993, Ogwang and Rao 1996). Furthermore, the Lorenz curve fulfills (Chotikapanich 1993)

$$0 < L(F(z)) < F(z) < 1. \tag{A.1}$$

A mathematical representation of the Lorenz curve can be derived as follows. Given an income distribution function with density function $f(x)$, the horizontal axis of the Lorenz curve is given by the cumulative distribution function

$$F(z) = \int_0^z f(x)dx \tag{A.2}$$

and the vertical axis is given by the first moment distribution function

$$L(F(z)) = \frac{1}{y}\int_0^z xf(x)dx = \frac{1}{y}\left[zF(z) - \int_0^z F(x)dx\right] \tag{A.3}$$

where y denotes the first moment of the density function f (i.e. the mean income/entitlement). The second identity has been derived by applying partial integration (see Atkinson 1970, Lam 1988). Differentiating equation (A.3) with respect to z yields

$$\frac{dL(F(z))}{dz} = \frac{1}{y}zf(z). \tag{A.4}$$

Furthermore, the following condition holds

$$\frac{dL(F(z))}{dz} = \frac{dL(F(z))}{dF(z)}\frac{dF(z)}{dz} = \frac{dL(F(z))}{dF(z)}f(z). \tag{A.5}$$

Combining equations (A.4) and (A.5) yields

$$\frac{dL(F(z))}{dF(z)}f(z) = \frac{1}{y}zf(z), \tag{A.6}$$

or equivalently

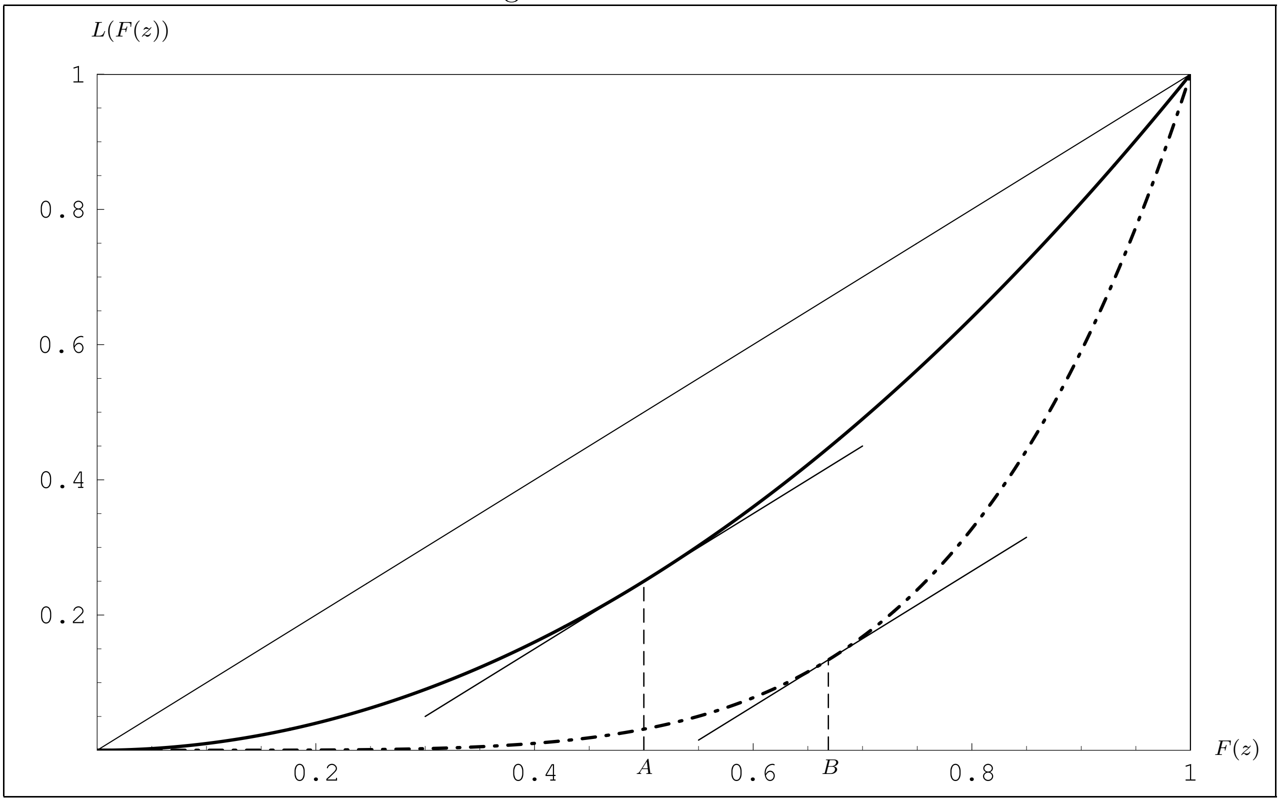
$$\frac{dL(F(z))}{dF(z)} = \frac{1}{y}z. \tag{A.7}$$

The last equality establishes the assertion that the slope of the Lorenz curve is inversely proportional to the mean income y.

Figure 6 plots Lorenz curves for various degrees of inequality in the food distribution, where the convexity of the Lorenz curve measures the degree of inequality which prevails in the economy, as proven below.

If the Lorenz curve coincides with the 45^o-line, the slope of the Lorenz curve equals one at each point. Hence, everyone in the economy would receive the mean income/entitlment, i.e. 'perfect equality' would prevail.

Figure 6: Lorenz curve



Recalling equation (A.7), we can determine the share of the population that receives an income/entitlement less than or equal to the mean income/entitlement in the economy for each Lorenz curve by marking the points on each Lorenz curve, where the slope equals one. The Lorenz curve which is denoted by the dotted line yields a higher percentage of population that receives income/entitlement less or equal to the mean income/entitlement (point B) than the Lorenz curve which is indicated by the solid line (point A). Consequently, the Lorenz curve that relates to the point B represents a higher degree of inequality. Obviously, this Lorenz curve is more convex.

A.2 Proof of Proposition 1

The computation of the steady states is constrained to the case $\tilde{y} > \alpha y$ since the transition function between the sub-population then simplifies to

$$M(Y_t, P_{It}, P_{St}) = P_{St}.$$

For $\tilde{y} < \alpha y$ the fixed points have to be determined numerically.

Taking

$$P_{I,t+1} = P_{It} = P_I \text{ and } P_{S,t+1} = P_{St} = P_S \quad \forall t$$

into account, equations (14)–(15) simplify to

$$(b_I - d(y_I))\, P_I + P_S = 0 \tag{A.8}$$

$$(b_S - d(y_S))\, P_S - P_S = 0. \tag{A.9}$$

Equation (A.9) yields that either the equilibrium number of food-secure population equals zero or that the natural population growth rate of the food-secure population equals one. For plausible parameter values one can abstain from the latter case.

Furthermore, Dworak (2002, chapter 5) demonstrates that there exists a unique negative relationship between the number of food-insecure population and the resource stock for a given number of food-secure population.

Moreover, the equilibrium number of the food-secure population equaling zero implies that either also the food-insecure population equals zero in the equilibrium or the natural population growth rate of the food-insecure equals zero from equation (A.8). However, the right-hand side of the difference equation system (14)–(15) is not defined for a zero total population size.

The condition of zero natural population growth for the food-insecure population implies that the per capita food entitlement of the food-insecure population equals the following expression

$$y_I(P_I, 0, R) = y_m + \frac{1}{\delta} \ln\left(\frac{d_{max}}{b_I - d_{nat}} - 1\right). \tag{A.10}$$

The equation above exhibits a unique solution for R taking also the negative relationship between R and P_I into account.

For the existence of the fixed point, the equilibrium values has to fulfill the condition $\tilde{y} > \alpha y(\hat{P}_I, 0, \hat{R})$. However, since the number of food-secure population equals zero, the per capita food entitlement of food-insecure population equals that of the total population i.e $y_I = y$. Hence, the condition above may be transformed to

$$\frac{\tilde{y}}{\alpha} > y_I = y_m + \frac{1}{\delta} \ln\left(\frac{d_{max}}{b_I - d_{nat}} - 1\right), \tag{A.11}$$

which is equal to

$$b_I - d(\tilde{y}/\alpha) > 0.$$

A.3 Technical Details of the Bifurcation Analysis

As evident from Figures 4–5, there exist either one or three fixed points. In general, the regions of the differing number of fixed points are separated by fold curves, which collide in a Cusp point. Furthermore, the stability of the fixed point changes and a closed invariant curve emerges when passing the Neimark-Sacker curves, which are abbreviated by 'NS' in Figures 4–5, where a '+' or '−' in the subscript denote whether the Neimark-Sacker bifurcation is subcritical or supercritical, respectively.[15] Furthermore, the fold and the Neimark-Sacker curve meet tangentially in a strong resonance bifurcation 1:1 (denoted by $R1:1$).[16]

However, at $\tilde{y} = \alpha y_I(\hat{P}_I, 0, \hat{R})$ which is equivalent to

$$n_I\left(\frac{\tilde{y}}{\alpha}\right) = b_I - d\left(\frac{\tilde{y}}{\alpha}\right) = 0,$$

the right-hand side of the equation system (14)–(16) is not differentiable. Hence, the bifurcation behavior cannot be inferred from the multipliers of the fixed point and, therefore, it has to be determined by observing orbits.

[15] At a supercritical Neimark-Sacker bifurcation the fixed point looses its stability and a stable closed invariant curve emerges. In contrast, at a subcritical Neimark-Sacker bifurcation, the fixed point becomes stable and an unstable closed invariant curve emerges.

[16] Furthermore, along the Neimark-Sacker curve an infinite number of Arnold tongues may be rooted. Moreover, according to Kutsnetsov (1998), there may exist transversal homoclinic structures in an exponentially narrow parameter region bounded by two smooth homoclinic tangencies curves. In addition, the fold curves delimiting the Arnold tongues accumulate on these homoclinic tangencies curve (Kutsnetsov 1998).

For instance passing from region ⓐ to ⓔ by crossing $n_I(\tilde{y}/\alpha) = 0$, the unique fixed point looses its stability and simultaneously an attracting closed invariant curve emerges, which is characteristic for a supercritical Neimark-Sacker bifurcation. Furthermore, passing from ⓑ to ⓓ, the stable fixed point with zero food-secure population and the unstable fixed point collide and vanish similar to a fold bifurcation. Moreover passing from ⓒ to ⓔ, it seems that both bifurcations occur, i.e. the stable and the unstable fixed points collide and vanish and a stable closed invariant curve appears.

Moreover, since the right-hand side of the equation system (14)–(16) is not differentiable everywhere, backwards iteration is not possible everywhere. Therefore, further unstable closed invariant curves may exist but cannot be detected or, if their existence is detected by subcritical Neimark-Sacker bifurcations, they cannot be continued as the parameters change.

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